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# On the application of fracture fatigue entropy to variable frequency and loading amplitude

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ABSTRACT

A nondestructive fatigue model is developed that utilizes the thermographic methodology and the concept of entropy production to predict the residual life of a component subjected to variable amplitude loading. The applicability of the model is investigated using a set of experiments on stainless steel 304 covering both low- and high-cycle fatigue regimes. Results are also presented that compare the predictions of the residual life with those obtained by applying the Miner's rule, quantitative thermographic methodology, fatigue driving stress, and the fatigue driving energy approaches. The results show that the maximum and average errors of the present approach are much lower than the above-mentioned methods. Also presented are the results of a series of variable-frequency fatigue experiments that are successfully predicted by the present methodology.

## 1. Introduction

The establishment of a predictive relationship between fatigue life and constant amplitude loading has been the subject of rich volumes of published works for many decades. Nevertheless, most mechanical components experience cyclic stresses with loading amplitudes that vary throughout their lifetime, and no widely accepted model is currently available for prediction of their fatigue life.

Historically, Miner is credited for having treated the problem of variable loading fatigue for the first time in 1945 by proposing the simplest cumulative damage model. Sometimes referred to as the "*Palmgren-Miner's rule*," this approach has become the basis for a number of other models that followed with different improved methodologies. Simply put, the Miner's rule states that if the component experiences $k$ different stress levels and the number of cycles to failure at the $i_{th}$ stress, $\sigma_i$, is $N_i$, then a cumulative damage variable, $D$, can be defined to represent the summation of individual damage fractions as follows [1].

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{1}$$

where $n_i$ is the number of cycles at a given stress amplitude, $\sigma_i$, and failure is assumed when $D$ approaches unity.

A well-known drawback of this model is its inability to take into account the effect of the loading sequence. Many reported experimental tests reveal that fatigue fracture occurs sooner if the initial stress level is high followed by a lower stress level (HL) compared to the same amplitude stress levels but in reverse order, i.e. changing the sequencing to low followed by high-stress (LH). Moreover, results of numerous published research reveal that the damage parameter can vary from $0.75 \le D \le 1.2$.

Many improvements to the Miner's rule have been proposed, notable among them is the non-linear stress-dependent cumulative damage theory by Marco-Starkey [2] that takes the effects of loading sequences into account. Based on Marco-Starkey, the damage parameter at a specific stress level is a variable quantity related to that stress amplitude. In this method, the damage path exponentially varies based on the amplitude of the applied stress.

In 2002 Fargione et al. [3] suggested a *quantitative thermographic methodology* to predict the residual fatigue life of components. According to their study, the integral thermal increment, ∅, up to the failure of the specimen is a constant value and can be obtained by the product of asymptotic temperature increment and the number of cycle to failure.

$$\varnothing \approx \frac{\Delta T_S N_s}{2} + \Delta T_S (N_f - N_s) \tag{2}$$

where $\Delta T_S$ is the temperature difference in the stabilized phase, $N_f$ stands for the number of cycles to failure, $N_s$ represents the total number of cycles to reach the stabilized temperature and ∅ is the integral of thermal increment up to the failure of the specimen. As noted in the literature [4–8] it has been shown that the total number of cycles

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**Nomenclature**

| | |
|---|---|
| $A$ | cross-sectional area of the gage section [$m^2$] |
| $b$ | constant |
| $c_p$ | specific heat capacity [$\frac{J}{kgK}$] |
| $D$ | damage parameter |
| $\dot{E}_{in}$ | energy rate entering the control volume [$\frac{W}{m^2}$] |
| $\dot{E}_{gen}$ | rate of internal heat source per unit time per unit volume [$\frac{W}{m^2}$] |
| $\dot{E}_{out}$ | energy dissipated rate from the control volume [$\frac{W}{m^2}$] |
| $f$ | load frequency [$\frac{Cycle}{s}$] |
| FFE | fatigue fracture entropy [$\frac{MJ}{m^3K}$] |
| $h$ | heat transfer coefficient [$\frac{W}{m^2K}$] |
| $n$ | number of cycle [*cycle*] |
| $N_f$ | number of cycle to failure [*cycle*] |
| $N_s$ | total number of cycles to reach the stabilized temperature [*cycle*] |
| $Q_{conv}$ | heat dissipated in a unit volume by convection [$\frac{MJ}{m^3s}$] |
| $Q_t$ | total energy generation per second [$\frac{MJ}{m^3s}$] |
| $Q_t^f$ | heat generation by internal friction [$\frac{MJ}{m^3s}$] |
| $Q_t^p$ | heat generation by microplastic deformation per second [$\frac{MJ}{m^3s}$] |
| $R_\theta$ | initial slope of surface temperature variation as a function of time [°C/s] |
| $R_\theta^f$ | portion of $R_\theta$ related to internal friction [°C/s] |
| $R_\theta^p$ | portion of $R_\theta$ related to microplastic deformation [°C/s] |
| $S$ | entropy production at any stage [$\frac{MJ}{m^3K}$] |
| $t$ | time [s] |
| $t_f$ | time to failure [s] |
| $T$ | temperature [K] |
| $T_s$ | steady-state temperature [K] |
| $T_\infty$ | room temperature [K] |
| $\dot{U}$ | change in stored energy [$\frac{W}{m^2}$] |
| $V$ | volume of the gage section [$m^3$] |
| $\Delta T$ | temperature rise at steady-state [K] |
| $\Delta T_S$ | thermal difference in the stabilized (quasi-isothermal) phase [K] |
| $\rho$ | density $\left[\frac{kg}{m^3}\right]$ |
| $\varepsilon_p$ | plastic strain |
| $\sigma$ | stress tensor[MPa] |
| $\varnothing$ | cumulative thermal increment to failure per unit volume $\left[\frac{K}{m^3}\right]$ |

to reach the steady-state temperature, $N_s$, is negligible in comparison with the total number of cycle to failure, $N_f$. Using this assumption, Eq. (2) reduces to:

$$\varnothing \approx \Delta T_S N_f \tag{3}$$

This definition enables one to formulate a damage accumulation rate expression analogous to that of the Miner's rule in the following form.

$$\sum_{i=1}^{n} \frac{\Delta T_{AS} n_i}{\varnothing} = 1 \tag{4}$$

In this formulation, each step in a variable loading experiment generates a part of the whole thermal increment.

In 2012 Kwofie and Rahbar [9] proposed a new modified Miner's rule formula called *fatigue driving stress approach* with the capability of predicting variable loading sequence of the following form.

$$\sum_{i=1}^{n} \frac{n_i ln(N_i)}{N_i ln(N_1)} = 1 \tag{5}$$

wherein $n_i$ is the number of accumulated cycles at the $i_{th}$ stage, and $N_i$ is the number of cycle to failure related to the stress at $i^{th}$ stage of the experiment. For the two-stage loading level, Eq. (5) can be defined as follow:

$$\sum_{i=1}^{2} \frac{n_i ln(N_i)}{N_i ln(N_1)} = \frac{n_1}{N_1} + \frac{n_2}{N_2}\frac{ln(N_1)}{ln(N_2)} = 1 \tag{6}$$

To predict the remaining life, Eq. (6) can be showed for $n_2$:

$$n_2 = N_2\left[\left(1-\frac{n_1}{N_1}\right)\frac{\ln(N_1)}{\ln(N_2)}\right] \tag{7}$$

When dealing with a low-to-high (LH) stress sequence, $N_1 > N_2$, resulting in $\frac{ln(N_1)}{ln(N_2)}\left(1-\frac{n_1}{N_1}\right) > \left(1-\frac{n_1}{N_1}\right)$, so $\sum_{i=1}^{n}\frac{n_i}{N_i} > 1$ and correspondingly for high-to-low (HL) case, $N_1 < N_2$, so that $\frac{ln(N_1)}{ln(N_2)}\left(1-\frac{n_1}{N_1}\right) < \left(1-\frac{n_1}{N_1}\right)$. Hence, $\sum_{i=1}^{n}\frac{n_i}{N_i} < 1$.

Peng et al. [10] used a combination of the fatigue driving stress and strain energy density. They proposed a new non-linear formula for the variable-amplitude loading applications, based on the so-called *fatigue driving energy approach*. For a simple two-level loading sequence with the first stress level $\sigma_1$ pertains to $n_1$ cycle and with $\sigma_2$ stress level, the second portion of life, $n_2$. Thus, the remaining life fraction, $\frac{n_2}{N_{f_2}}$, can be obtained by the following non-linear formula:

$$\frac{n_2}{N_{f_2}} = 1-\frac{1}{-2bln(N_{f_2})}ln(N_{f_2}{}^{-2b}-1)\left(\left(\frac{N_{f_2}{}^{-2b\frac{n_1}{N_{f_1}}}-1}{N_{f_2}{}^{-2b}-1}\right)^{\frac{\sigma_2}{\sigma_1}}+1\right) \tag{8}$$

In order to consider the effect of the loading history on the damage accumulation under such conditions, the load interaction effects are incorporated into this model. It is worth mentioning that the most of these methods do not take into account the effect of operating frequency for predicting remaining life.

In the present work, we investigate the feasibility of utilizing the concept of thermodynamic entropy generation in a fatigue process to predict the remaining life under variable loading experiments with consideration of loading sequencing. The approach is based on the concept of fatigue fracture entropy (FFE) which asserts that the cumulative entropy up to the point of fracture is a material property [11,12]. To illustrate the applicability of the approach, the results of a series of variable loading experiments are reported that cover both low- and high-cycle fatigue. Also presented are model verification based on experimental results conducted at different operating frequencies.

## 2. Theory

The damage accumulation theory asserts that the eventual fracture of a specimen is due to progressive microplastic deformations and the consecutive formation of microcracks [13–17]. These are irreversible damage processes with the associated dissipation of energy. In addition, there exists energy dissipation due to internal friction which does not cause fatigue damage. Internal friction happens due to the oscillation of grain boundaries, dislocations, and atoms. In the case of low-cycle fatigue (LCF), the non-damaging part of dissipation energy is negligible and the total dissipation is nearly equal to the dissipation due to the microplastic deformations. In the case of high-cycle fatigue (HCF), however, both damaging and non-damaging parts are active participants and must be properly accounted for. Fig. 1 presents a schematic illustration of how the total dissipation energy varies with stress. The stress beyond which the irreversible deformation mechanisms become