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A simple explicit homogenization solution for the macroscopic elastic response of isotropic porous elastomers

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Abstract

An approximate homogenization solution is put forth for the effective stored-energy function describing the macroscopic elastic response of isotropic porous elastomers comprised of incompressible non-Gaussian elastomers embedding equiaxed closed-cell vacuous pores. In spite of its generality, the solution — which is constructed in two successive steps by making use first of an iterative technique and then of a nonlinear comparison medium method — is fully explicit and remarkably simple. Its key theoretical and practical features are discussed in detail and its accuracy is demonstrated by means of direct comparisons with novel computational solutions for porous elastomers with four classes of physically relevant isotropic microstructures wherein the underlying pores are: (*i*) infinitely polydisperse in size and of abstract shape, (*ii*) finitely polydisperse in size and spherical in shape, (*iii*) monodisperse in size and spherical in shape, and (*iv*) monodisperse in size and of oblate spheroidal shape.

Key words: elastomers, microstructures, porosity, constitutive modeling, Hamilton-Jacobi equations

1. Introduction and main result

The prime objective of this paper is to construct an approximate homogenization solution for the effective stored-energy function describing the macroscopic elastic response of isotropic porous elastomers at arbitrary finite deformations.

Microscopic description of the porous elastomers of interest. More specifically, our focus in this work is on isotropic porous elastomers that comprise a statistically uniform and isotropic distribution of equiaxed — but of arbitrary shape otherwise — closed-cell vacuous pores embedded in an isotropic incompressible non-Gaussian elastomeric matrix. In its ground state, presumed to be undeformed and free of stress, the bounded domain in $\mathbb{R}^3$ occupied by any such porous elastomer is denoted by Ω_0 and its boundary by $\partial\Omega_0$. The largest characteristic size of the underlying pores is taken to be much smaller than the size of Ω_0 and, for convenience, units of length are chosen so that Ω_0 has unit volume. The constitutive behavior of the underlying elastomeric matrix is characterized by a free-energy density $W_{\mathtt{m}}$, which is taken to be an objective and isotropic function of the deformation gradient tensor $\mathbf{F}$, in the sense that $W_{\mathtt{m}}(\mathbf{QFK}) = W_{\mathtt{m}}(\mathbf{F})$ for all $\mathbf{Q}, \mathbf{K} \in Orth^+$ and arbitrary $\mathbf{F}$. In particular, we shall consider non-Gaussian free- or stored-energy functions of the form

$$W_{\mathtt{m}}(\mathbf{F}) = \begin{cases} \Psi_{\mathtt{m}}(I_1) & \text{if} \quad J = 1 \\ \\ +\infty & \text{otherwise} \end{cases}, \tag{1}$$

where $I_1 = \mathbf{F}\cdot\mathbf{F} = F_{ij}F_{ij}$, $J = \det\mathbf{F}$, and where $\Psi_{\mathtt{m}}$ stands for any non-negative function of choice satisfying the linearization conditions[1] $\Psi_{\mathtt{m}}(3) = 0$ and $\Psi'_{\mathtt{m}}(3) = \mu/2$, with μ denoting the initial shear modulus of the

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[1] Throughout we make use of the standard notation $y'(x) = \mathrm{d}y(x)/\mathrm{d}x$.