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Detection of symmetry breaking bifurcations using finite element analysis packages

Iqbal Alshalal, Z.C. Feng *

Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211, United States

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ABSTRACT

Trusses with geometric and loading symmetries have been used in many structures to reduce the complexity of the design. Slight asymmetries in geometry and in loading could lead to bifurcation in the structural response. Failure of such structures still occur occasionally causing major damage to the property and to the human lives. A fully nonlinear structural analysis is expected to detect such symmetry breaking bifurcations. In this paper, we conducted the bifurcation analysis of a two-bar truss and a shallow arch structure with seven bars. Two program packages Gesa and Ansys based on finite elements method have been used to detect the symmetry breaking bifurcation points. However, unlike typical bifurcation analysis packages they cannot detect the bifurcation point without inserting a small perturbation in the initial geometry or the loading of the structure. Such a bifurcation can be easily missed in finite element-based analysis. The theoretical analysis reveals that the bifurcation leads to a much lower critical load in the presence of small asymmetry compared to the symmetric case. The results are verified by using Gesa program in Matlab for fully nonlinear analysis and with results by using Ansys commercial program. The two structural examples serve to illustrate the limitations of widely used finite element analysis packages for nonlinear bifurcation analysis.

1. Introduction

A structure's response under static load is often described by the load deflection curve. Each point in the curve corresponds to an equilibrium state in the equilibrium path. A bifurcation point is the point on the load deflection curve which has two or more directions to go when the load varies [1], and it is one of the critical points in the equilibrium path when the system changes from being stable to unstable [2]. Instability resulting from bifurcations is an important failure mode in many thin walled or skeletal structures. A structure is said to have snapped when the equilibrium path emerging from unloaded state to a loaded state loses its stability and reaches to another more stable point. Eventually, the structure reaches a new stable configuration [3]. A primary stable equilibrium path rising with load parameter cannot become unstable unless it intersects with another secondary path at a bifurcation point [4]. However, the system may lose stability at a limit point bifurcation. It can be said that the system will snap towards a far stable equilibrium position and the structure buckles once it reaches the bifurcation point at the maximum loading. The snap-through nonlinear deformation of an inflated balloon and its growth involving both primary and bifurcated branches have been explored and studied numerically using finite element method [5].

The main reason for system instability is system nonlinearity. The nonlinearity sources in solid mechanics are: force nonlinearity, kinematic nonlinearity, material nonlinearity, and geometrical nonlinearity. The bifurcation point was detected by using a geometrically nonlinear analysis [6,7]. The geometrical nonlinearity behavior is often seen in slender structures such as arches, trusses and membranes [8,9,5]. Researchers have suggested using a truss to learn about the nonlinear geometric behavior [10]. Slenderness and shallowness ratios are two parameters that are important in the interaction between snap-through and Eulerian instability. The shallowness ratio is the height to length ratio which may also be quantified by the initial angle of the arch [9]. The slenderness ratio is the ratio between the length of a structural element and the thickness of the element. Multiple-short truss elements are used in the nonlinear analysis procedure to model the bent effects of long cables [11]. Different methods to predict and solve the problem of bifurcation analysis of a structure have been used and presented in the literature. The analytical solution is very important to clarify the fundamentals of nonlinear behavior. Such a solution is used in the current study with a Von Mises truss which is a simple truss with two pinned bars [12]. A few analytical solutions are offered for severe geometrically nonlinear behaviors [13,14]. Many researchers

* Corresponding author.
E-mail addresses: ia423@mail.missouri.edu (I. Alshalal), fengf@missouri.edu (Z.C. Feng).

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have used simple structures in their studies to explain the basic concepts of stability behavior [15]. A simple example of two-bar planar truss has been analyzed to illuminate the snap-through and the bifurcation behavior of the structures [16]. Simple pinned–pinned beam (initially flat), formulated based on classical Euler–Bernoulli theory has been used to study the relation between Euler buckling and the instability phenomena [17].

Finite element methods with the assumption of the geometrical nonlinearity have been used to solve stability problem of Von Mises truss [18]. Some structures show highly geometrically nonlinear responses that exhibit complex snap-back behavior. These responses can be modeled using the finite element approach with employment of the arc-length method to analyze the nonlinear equilibrium path through limit points for three-dimensional space trusses [19]. Integral equation method with conjugate arc method can follow the nonlinear equilibrium path in shallow arches and overcoming bifurcation and limit points [20]. Some structures can exhibit snap-through buckling behavior under lateral loading. This behavior is reveled in curved structures, such as panels, beams and arches [21].

The snap-through and snap-back phenomena are very common and dominant in highly flexible truss structures. They eventually lead to large deformation in the truss structures [1]. The field of deployable structures has many potential applications in space for their advantages of small volume that can be occupied and its highly nonlinear geometry which depends on snap-through instability type behavior of the structure [22,23].

It was noted from literature that the instabilities are associated with limit and bifurcation points. A structure can lose the stability before reaching the theoretical ultimate load. The judgment of stability can be evaluated by finding the determinant of the stiffness matrix for the system extracted from first order derivative of potential energy. The instability takes place when the determinant of the matrix is negative [24]. The concept of catastrophe theory has been utilized to find the cusp point singularity using third order derivative of the potential energy function [25]. In many practical cases, instability can be detected by checking two or more eigenmodes at coincident or nearly coincident critical loads. This coupled instability might give rise to severe imperfection sensitivity [26]. An instability phenomenon can be avoided through special design modification; however, in recent studies, researchers have been attempting to transform the negative effect to the positive one, as a beneficial behavior to be used in the design of smart devices. Based on the changes in the shape of the structure concept, buckling is induced in micro-electromechanical systems (MEMS) devices [27]. Many MEMS devices have been manufactured with low production cost and with advanced and integrated circuit technologies relying on the benefits of the buckling [28].

In this study, the structural material is assumed to be linear-elastic [29] and only geometrical nonlinearity is considered. In a prior study, geometrical imperfection was introduced for the Von Mises arch as a deviation in midspan of each beam from the line axis in two patterns. The first used symmetric imperfection where the deviation applied downward in each beam. The second reverses direction for the adjacent beams as an asymmetric imperfection [30]. However, in our study, small load and displacement perturbation in the initial conditions are used as asymmetric perturbation. In our study, asymmetric perturbation is used to detect the bifurcation. We carried out the bifurcation analysis of two structures. Due to the simplicity of these structures, stability criteria can be obtained analytically. Under asymmetry perturbations, a much lower critical load was found for each structure. Using the same two structures, we then demonstrate the use of two packages based on finite element methods for the bifurcation analysis. Although the same critical loads are obtained, the analysis using FEM is more suitable for complex structures. However, care must be taken when using these packages; the computer programs may fail to detect some bifurcations unless an asymmetric perturbation, such as the loading or in the geometry, is inserted. The newly detected bifurcations of the two truss structures are important since the critical loads at these bifurcations are much lower than the ultimate load corresponding to the limit point of an arch. We have presented parametric study for a truss with different initial angles. The detected bifurcations may also provide designers novel options when utilizing nonlinear structural responses in devices.

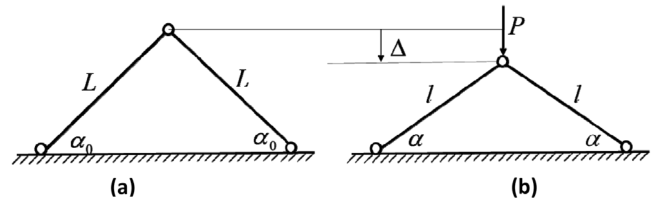

Fig. 1. Symmetric truss supporting load.

2. Symmetry breaking bifurcation of a Von Mises arch

Two assumptions are considered throughout our analysis. First, the Von Mises arch is linearly elastic; geometric nonlinearity refers to large nodal displacement and moderate axial strains (<0.05). Second, high Euler buckling is assumed to neglect the possibility of buckling instability [25,31]. On the assumption of moderate strain, we found that the difference between Cauchy strain and Green strain is (0.027%), which is insignificant. Fig. 1 shows an arch, traditionally named Von Mises arch, with two elastic hinged members under a compressive load. We include the elastic deformation in the equilibrium equation and treat the two compressive members as springs with spring constant $k = \frac{EA}{L}$, where E, A and L are the Young's modulus, the cross sectional area, and the unloaded length of each compressive member, respectively.

2.1. Snap-through bifurcation and the ultimate load limit of the symmetric structure

Let Δ denote the downward displacement of the top node, α_0 the initial angle, and α the angle after deformation. Fig. 1 with trigonometric relationships leads to the followings:

$$\sin\alpha = (L\sin\alpha_0 - \Delta)/l \tag{1}$$

$$L\cos\alpha_0 = l\cos\alpha \tag{2}$$

The force P is calculated using the force in the truss based on:

$$\frac{P}{sin\alpha} = 2k(L-l) \tag{3}$$

Substituting (2) into (3), we have

$$\frac{P}{kL} = 2\sin\alpha\left(1 - \frac{\cos\alpha_0}{\cos\alpha}\right) \tag{4}$$

$$sin\alpha = \frac{Lsin\alpha_0 - \Delta}{\sqrt{(Lcos\alpha_0)^2 + (Lsin\alpha_0 - \Delta)^2}}, \quad \cos\alpha = \frac{Lcos\alpha_0}{\sqrt{(Lcos\alpha_0)^2 + (Lsin\alpha_0 - \Delta)^2}} \tag{5}$$

Substituting (5) into (4), we get

$$\frac{P}{kL} = 2[(Lsin\alpha_0 - \Delta)(L^2 - 2\Delta Lsin\alpha_0 + \Delta^2)^{-\frac{1}{2}} - \frac{Lsin\alpha_0 - \Delta}{L}] \tag{6}$$

This can be written in the following dimensionless form:

$$\frac{P}{kL} = 2(\sin\alpha_0 - \frac{\Delta}{L})\left[(1 - 2\frac{\Delta}{L}\sin\alpha_0 + (\frac{\Delta}{L})^2)^{-1/2} - 1\right] \tag{7}$$

The relationship between dimensionless load (P/kL) and the dimensionless displacement (Δ/L) is nonlinear as seen from Eq. (7) and it is shown in Fig. 2 for $\alpha_0 = 20°$ and $\alpha_0 = 15°$. Both curves snap-through at the critical points (limit points) to another stable path. Larger

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