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Experimental validation of model predictive control stability for autonomous driving☆

Pedro F. Lima [a,b,*], Gonçalo Collares Pereira [a,b], Jonas Mårtensson [a], Bo Wahlberg [a]

[a] Integrated Transport Research Lab, Department of Automatic Control, School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden
[b] Research and Development, Scania CV AB, 151 87, Södertälje, Sweden

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ABSTRACT

This paper addresses the design of time-varying model predictive control of an autonomous vehicle in the presence of input rate constraints such that closed-loop stability is guaranteed. Stability is proved via Lyapunov techniques by adding a terminal state constraint and a terminal cost to the controller formulation. The terminal set is the maximum positive invariant set of a multi-plant description of the vehicle linear time-varying model. The terminal cost is an upper-bound on the infinite cost-to-go incurred by applying a linear–quadratic regulator control law. The proposed control design is experimentally tested and successfully stabilizes an autonomous Scania construction truck in an obstacle avoidance scenario.

1. Introduction

Autonomous vehicles will inevitably face emergency situations, in which they may need to maneuver aggressively to avoid, for example, an imminent collision. Road traffic injuries, 94% of which are caused by human error (European Commission, 2011), are predicted to become the third most common cause of disability by 2020 (World Health Organization, 2009). Thus, when removing the human-factor from the equation, ensuring vehicle stability during safety-critical events is of utmost importance when developing commercial autonomous vehicles. Verified stability is a key aspect for safe and reliable autonomous vehicles.

The road to autonomous driving has been slowly paved with the gradual introduction of advanced driving assistant systems (e.g., anti-lock braking system, electronic stability control, adaptive cruise control, lane departure warning system, and automatic parking). These systems play a major role of support to the driver both in critical and tedious situations, reducing the number of traffic accidents and fatalities (Ross, 2014). In particular, ESC intervenes when the steering command given by the driver yields an unstable vehicle motion. Nevertheless, the design of the motion controller module for autonomous vehicles must attain a stable behavior and cannot rely on an eventual unstable behavior being avoided by driving assistance technology. This work addresses the problem of designing motion controllers for autonomous driving that ensure closed-loop stability. In particular, Model Predictive Control (MPC) closed-loop stability issues in practice are investigated. The results show that a standard MPC, without properly designed terminal cost and terminal state set, can lead to instability, particularly when the prediction horizon is short. On the one hand, ensuring vehicle stability facilitates controller certification and standardization when entering the development phase. On the other hand, shortening the prediction horizon gives room to more computational demanding modules. In the first attempts to demonstrate autonomous driving, in the DARPA challenges, the focus was rather in developing a feasible system architecture than in testing the stability limits of the vehicles (Thrun, Montemerlo, Dahlkamp, Stavens, Aron, Diebel, et al., 2006; Urmson, Bagnell, Baker, Hebert, Kelly, Rajkumar, et al., 2007). Even recent vehicle development assume that the vehicle operation is well below handling limits and few or no attention is given to situations of possible vehicle instability (Mchugh, 2015; Ziegler, 0000; Ziegler, Bender, Schreiber, Lategahn, Strauss, Stiller, et al., 2014).

The motion controller is a crucial module in the design of an autonomous vehicle as it is responsible for stabilizing and guiding the vehicle along a given reference path. In the recent decades, MPC has gained increasing attention to address the problem of vehicle control. With the increase of computational power and optimization solvers efficiency, MPC has become quite popular, since it handles nonlinear

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* Corresponding author at: Integrated Transport Research Lab, Department of Automatic Control, School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden.

E-mail addresses: pfrdal@kth.se (P.F. Lima), gpcp@kth.se (G. Collares Pereira), jonas1@kth.se (J. Mårtensson), bo@kth.se (B. Wahlberg).

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time-varying models and constraints in a systematic manner. Using a model to predict the system behavior, an user-defined cost function is minimized, and the optimal sequence of inputs is computed in order to follow a specified path or trajectory under known constraints on states and inputs. A subset of the optimal input sequence is applied to the vehicle and the process is then repeated (Bemporad, 2006; Garcia, Prett, & Morari, 1989; Mayne, Rawlings, Rao, & Scokaert, 2000). One of the strengths of MPC is the possibility of explicitly include additional constraints and cost terms, which lead to closed-loop stability guarantees. One of the most popular strategies for ensuring closed-loop stability using MPC (see Mayne et al., 2000 and references therein) is to use the optimization value function as a Lyapunov function. Moreover, the analysis is convenient if incorporating both a terminal cost and a terminal state set in the optimal control problem. The terminal cost is chosen such that it is equal to the infinite-horizon value function in a suitable neighborhood of the origin (i.e., the terminal state set). Hence, it is possible to use the known advantages of an infinite-horizon control, such as guaranteed stability (Keerthi & Gilbert, 1988).

The closed-loop stability properties when using MPC have been extensively studied from the theoretic point of view (see Mayne et al., 2000 and references therein). However, the lack of implementability of many of the proposed control designs makes the practical analysis less frequent in the literature. When experimental evaluation is considered, the large majority of the works either leave the stability concerns out (Thrun et al., 2006; Urmson et al., 2007), or the vehicle *looks* stable due to careful tuning of controller (Lima, Trincavelli, Mårtensson, Nilsson, and Wahlberg, 2017; Liniger, Domahidi, & Morari, 2015; Turri, Carvalho, Tseng, Johansson, & Borrelli, 2013) or due to the inclusion of vehicle dynamics constraints (Beal & Gerdes, 2013; Falcone, Borrelli, Asgari, Tseng, & Hrovat, 2007; Falcone, Borrelli, Tseng, Asgari, & Hrovat, 2008a; Funke, Brown, Erlien, & Gerdes, 2017; Katriniok, Maschuw, Christen, Eckstein, & Abel, 2013). The scenarios presented range from lane-keeping and obstacle avoidance (Funke et al., 2017; Turri et al., 2013), to racing applications (Beal & Gerdes, 2013; Liniger et al., 2015). Moreover, the presented experiments typically consider low-friction roads (Falcone et al., 2007; Turri et al., 2013) or vehicle handling-limits (Funke et al., 2017; Katriniok et al., 2013). Stability is ensured by constraints that bound the tire slip angle. Consequently, the vehicle motion is bounded within the region of the state space that does not contain unstable vehicle dynamics. Common to all of these schemes is the absence of explicit stability-imposing constraints in the MPC formulations. An exception is (Falcone, Borrelli, Tseng, Asgari, & Hrovat, 2008b), where a stability condition is proposed for a Linear Time-Varying (LTV) MPC scheme used in active front steering systems. An additional convex constraint bounding a quadratic function of the control effort and the predicted states is computed to ensure stability. However, this requires the MPC to be cast as a Sequential Quadratic Program (SQP) that has typically higher computational burden than a QP. Additionally, simplifications are made, such as considering the model time-invariance by linearizing around the current set point and assuming that the terminal state set is a singleton. Although this reduces the complexity of the overall design, it also affects the feasibility region of the controller.

1.1. Main contributions

The main contributions of this paper are:

1. the offline computation of the terminal cost and terminal state set for linear time-varying model predictive controller (LTV-MPC) closed-loop stability;
2. the proof of LTV-MPC closed-loop stability using the novel terminal cost and terminal state set;
3. the interpretation of the MPC parameter tuning influence in the design of the terminal cost and terminal state set in the autonomous driving case;
4. the effectiveness of the proposed control design in an autonomous Scania construction truck in simulation and experimentally.

The work presented here is an extension of Lima, Mårtensson, and Wahlberg (2017), where closed-loop stability is proved when using and LTV-MPC to lateral control an autonomous truck. There, a nonlinear kinematic vehicle model is linearized around a reference path, yielding an LTV model. The vehicle is modeled in the spatial domain and in a road-aligned coordinate frame with respect to a reference path to exclude time and speed from the dynamics equations (Frasch, Gray, Zanon, Ferreau, Sager, Borrelli, et al., 2013; Gao, Gray, Frasch, Lin, Tseng, Hedrick, et al., 2012; Lima, Oliveira, Mårtensson, Bemporad, and Wahlberg, 2017; Plessen, Lima, Mårtensson, Bemporad, & Wahlberg, 2017; Verschueren, De Bruyne, Zanon, Frasch, & Diehl, 2014). That work used the notion of multi-plant description (Badgwell & Thomas, 1997; Kothare, Balakrishnan, & Morari, 1996), in which the LTV model is divided in several Linear Time-Invariant (LTI) models. Moreover, it proposes that the maximum positive invariant set of over all the LTI models in the multi-plant description would be the terminal state set. The terminal cost was computed solving a convex min–max optimization problem (Lu & Arkun, 2000) that leads to the determination of the *worst* time-invariant model if used as a prediction model.

In this work, the notions of multi-plant description and space-based road-aligned vehicle model are used again. In addition to Lima and Mårtensson et al. (2017), this work

1. presents a new approach for computing the terminal cost;
2. proposes terminal cost and terminal state set considering input rate constraints;
3. shows the experimental evaluation of the proposed control design.

In this paper, the terminal cost is proposed to be the upper-bound on the cost-to-go incurred by applying a Linear–Quadratic Regulator (LQR) control law to any of the possible models in the multi-model representation. An upper-bound can be obtained by positively scaling one of Riccati matrices resulting from the cost-to-go calculation, which considers that the vehicle model is contained inside a convex uncertainty polytope. Moreover, including input rate constraints adds one extra dimension to both the terminal state set and weight matrix. This extra dimension is the input and it plays a major role in ensuring vehicle stability. With the proposed cost and state set design, stability and feasibility of the proposed LTV-MPC scheme are theoretically proved. The MPC parameter tuning influence is discussed in the light of autonomous driving. The effectiveness of the proposed MPC design is evaluated in simulation and in real experiments with a construction Scania truck in a scenario that resembles an emergency maneuver, where the vehicle avoids a fictitious obstacle. The controller successfully stabilizes an autonomous Scania construction truck even when other controllers, with no or milder terminal cost and terminal state set, cannot do it.

The development of the methods presented in this paper has in mind their practical applicability. Therefore, including the terminal cost and terminal state set in the control design should neither affect the execution time nor the optimization convexity. However, offline-computed solutions may be conservative as they need to cover a larger set of scenarios *a priori*, rather than computing less conservative terminal cost and terminal state set online.

1.2. Outline

The remainder of this paper is organized as follows. Section 2 introduces theoretical preliminaries; Section 3 addresses the problem of reference tracking using a receding-horizon framework by developing an LTV-MPC controller; Section 4 presents the nonlinear space-based vehicle model in the road-aligned coordinate frame used in the autonomous driving example. Furthermore, the same section proposes the terminal cost and terminal state set, which are used for proving