Automatica 98 (2018) 103–113

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

# Exponential convergence under distributed averaging integral frequency control ☆

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ARTICLE INFO

*Article history:*
Received 14 June 2017
Received in revised form 7 February 2018
Accepted 3 August 2018

*Keywords:*
Lyapunov methods
Networked systems
Power networks
Robustness analysis
Cyber–physical systems

ABSTRACT

We investigate the performance and robustness of distributed averaging integral controllers used in the optimal frequency regulation of power networks. We construct a strict Lyapunov function that allows us to quantify the exponential convergence rate of the closed-loop system. As an application, we study the stability of the system in the presence of disruptions to the controllers' communication network, and investigate how the convergence rate is affected by these disruptions.



## 1. Introduction

Modern power grids can be regarded as a large network of control areas, each producing and consuming power and transferring it to adjacent areas. The frequency of the AC signal is tightly regulated around its nominal value of e.g. 50 Hz to guarantee reliable operation of this network. Traditionally, this is achieved by means of proportional ('droop') control and PI control. In this setup, each area compensates for its local fluctuations in load, and adjusts its production to provide previously scheduled power flows to the adjacent areas. As a result, estimates of the load in each area are required in advance to achieve economical efficiency.

Recently, renewable energy sources such as wind turbines have been introduced in significant numbers. Since these sources do not usually provide a predictable amount of power, the net load on the individual control areas will change more rapidly and by larger amounts. More substantial fluctuations are expected to occur in microgrids, which are energy systems that can operate independently of the main grid. The resulting need for more advanced control strategies for future power networks has led to the design of distributed controllers equipped with a real-time communication network (Bürger & De Persis, 2015; Dörfler, Simpson-Porco, & Bullo, 2016; Mojica-Nava, Macana, & Quijano, 2014; Shafiee, Guerrero, & Vasquez, 2014; Trip, Bürger, & De Persis, 2016).

The addition of a communication network raises a reliability and security problem, as communication packets can be lost and digital communication networks may fall victim to failures and malicious attacks. A common disruption is the so-called Denial of Service, or DoS (Byres & Lowe, 2004), which can be understood as a partial or total interruption of communications. It is therefore of interest to characterize the performance degradation of the aforementioned networks of distributed controllers under loss of information, possibly due to a DoS event.

### 1.1. Literature review

The current research on frequency regulation in power networks is reviewed in Ibraheem, Kumar, and Kothari (2005). Since this field of research receives considerable amounts of attention, we will summarize a subset of results that are close to our interest.

Frequency stability and control in power networks is a well-established field of research which has lead to important results for a variety of models (see e.g. Bergen & Hill, 1981; Tsolas, Arapostathis, & Varaiya, 1985). More recently, distributed control methods have been proposed to guarantee not only frequency regulation but also economic optimality. In a microgrid context,

☆ This work was partially supported by the NWO-URSES project ENBARK (Grant number is 408-13-037), the DST-NWO IndoDutch Cooperation on "Smart Grids – Energy management strategies for interconnected smart microgrids" and the STW Perspectief program "Robust Design of Cyber–physical Systems" – "Energy Autonomous Smart Microgrids". The material in this paper was partially presented at the 20th World Congress of the International Federation of Automatic Control, July 9–14, 2017, Toulouse, France (Weitenberg, De Persis & Monshizadeh, 2017). This paper was recommended for publication in revised form by Associate Editor Yang Shi under the direction of Editor Thomas Parisini.

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https://doi.org/10.1016/j.automatica.2018.09.010
0005-1098/

distributed averaging integral control is well-studied (Andreasson, Tegling, Sandberg, & Johansson, 2017; Bürger & De Persis, 2015; Dörfler et al., 2016; Simpson-Porco, Dörfler, & Bullo, 2013; Trip et al., 2016). In the context of power networks, distributed internal-model-based optimal controllers have also been studied (Bürger & De Persis, 2015; Trip et al., 2016). As a complementary approach to distributed integral or internal-model controllers, primal–dual gradient controllers (Li, Chen, Zhao, & Low, 2014; Mallada, Zhao, & Low, 2017; Stegink, De Persis, & van der Schaft, 2017; Zhang & Papachristodoulou, 2013) are able to handle general convex objective functions as well as constraints, but in turn require much information about the power network parameters.

The robustness of power networks under various controllers has been investigated in the works above to varying degree. In this light, it is useful to consider strictly decreasing energy functions (Malisoff & Mazenc, 2009). Zhao, Mallada, and Dörfler (2015) make a first attempt to arrive at one, and their effort is expanded upon by Schiffer, Dörfler, and Fridman (2017) in the context of time-delayed communication. Bearing this in mind, we propose a construction of a new strict Lyapunov function for the purpose of explicitly quantifying the *exponential* convergence of power networks under distributed averaging integral control and then study the performance of this control in the presence of communication disruptions.

As an application of robustness measures, we will investigate the effect of Denial of Service. It, and related phenomena, have been studied as well. See e.g. Byres and Lowe (2004) for an introduction to the subject. It can be modeled as a stochastic process (Befekadu, Gupta, & Antsaklis, 2015), a resource-constrained process (Gupta, Langbort, & Başar, 2010), or using only constraints on the proportion of time it is active (De Persis & Tesi, 2014, 2015). Correspondingly, the investigations of systems under DoS events vary, with focus being on planning transmissions outside the disruption intervals (Shisheh Foroush & Martínez, 2013), limiting the maximum ratio of time during which DoS is active (De Persis & Tesi, 2015), or guaranteeing stability regardless and quantifying convergence behavior (De Persis & Tesi, 2014, 2015). The latter approach offers interesting perspectives, since the specific characterization of the period of time during which communication is not permitted adopted in De Persis and Tesi (2014) allows for great flexibility and can conveniently model both genuine loss of communication or packet drops due to malicious behavior. Furthermore, the analysis of De Persis and Tesi (2014, 2015) is based on Lyapunov functions, can handle distributed systems (Senejohnny, Tesi, & De Persis, 2015; Senejohnny, Tesi, & De Persis, 2017), and therefore is well suited for the class of nonlinear networked models describing power networks.

### 1.2. Main contribution

The contribution of this paper is primarily theoretical: existing approaches to the problem of optimal frequency control have mostly relied on non-strictly decreasing energy — or Lyapunov functions, using LaSalle's invariance principle and related results to guarantee convergence to an invariant manifold on which the Lyapunov function's derivative vanishes (see Schiffer et al., 2017; Vu & Turitsyn, 2017 for exceptions). Since this does not lead to strong results on convergence, we design a strictly decreasing Lyapunov function that does prove exponential convergence to the optimal synchronous solution. Our primary motivation for investigating this property is to provide an analytical tool with which robustness of the closed-loop system to disruptions can be quantified. Additionally, the Lyapunov function proposed in this paper is useful for analysis of related systems, as exemplified by Weitenberg et al. (2018).

As an illustration, the final part of the paper makes use of the developed Lyapunov function to show exponential convergence to the optimal solution in spite of possible communication interruptions, modeled here as complete temporary removal of the communication network. This is a simplification of the many possible scenarios that could occur (see Remark 6). We directly relate the speed of convergence to the physical parameters of the system and the availability of the communication network. As a result, the resilience of the aforementioned economically optimal control strategies to DoS events is quantified explicitly.

The remainder of this paper is organized as follows. In Section 2, we outline our model for the power network, goals for its control, and existing control strategies we will use. Then, in Section 3, we derive a strictly decreasing Lyapunov function and show exponential convergence to the optimal solution. In Section 4, we introduce a model for communication disruptions, and use our Lyapunov function to study the robustness of distributed controllers to these disruptions. In Section 5, we illustrate the main result using numerical simulations of an academic model of a power network. Finally, Section 6 presents conclusions.

### 1.3. Notation

Given a system state $x = x(t)$, we use the notation $\dot{x}$ to mean the time derivative $\frac{\partial x}{\partial t}$. Likewise, a function $f : \mathbb{R}^n \to \mathbb{R}$ of such a state, such as a Lyapunov function, has time derivative $\dot{f} := (\nabla_x f(x))^\top \dot{x}$. We denote its Hessian by $\nabla^2 f$. When used with vector arguments, sin and cos are defined element-wise. The symbols and $\mathbb{1}$ denote vectors and matrices filled with 0 and 1 respectively; if there is ambiguity about their size, the dimensions are given as a subscript. Finally, $\text{sp}(A) := \frac{1}{2}(A + A^\top)$ is used to denote the symmetric part of a square matrix $A$.

## 2. Setting

We consider a power grid, represented here by a set of $n$ buses. The network of power lines between the buses is represented by a connected graph with $n$ nodes and $m$ arbitrarily oriented edges and with $\pm 1$-valued incidence matrix $\mathcal{B}$. The orientation is necessary for analytical purposes but otherwise meaningless; the physical network is undirected.

We will use a structure-preserving model for the power network. We consider two types of nodes. Some nodes in the network are connected to synchronous generators or inverters with filtered power measurements; these we call generators. The others, which we will refer to as loads, are frequency-responsive loads or inverters with instantaneous power measurements and primary droop control. In this work, we disregard the additional possibility of 'passive' nodes that do not contribute to frequency control at all. Accordingly, we define the sets **G** and **L** of generator and load nodes with cardinality $n_{\mathbf{G}}$ and $n_{\mathbf{L}}$ respectively, such that $n_{\mathbf{G}} + n_{\mathbf{L}} = n$.

The dynamics at each bus is considered in a reference frame that rotates with a certain nominal frequency, i.e. 50 Hz. The dynamics can be expressed in the following form, also known as the swing equations (Kundur, Balu, & Lauby, 1994). At generator node $i \in \mathbf{G}$,

$$\dot{\theta}_i = \omega_i \tag{1a}$$

$$M_i \dot{\omega}_i = -D_i \omega_i - \sum_{j \in \mathcal{N}_i} \gamma_{ij} \sin(\theta_i - \theta_j) + u_i - P_i, \tag{1b}$$

whereas at load node $i \in \mathbf{L}$,

$$0 \quad = -D_i \omega_i - \sum_{j \in \mathcal{N}_i} \gamma_{ij} \sin(\theta_i - \theta_j) + u_i - P_i, \tag{1c}$$

Here, $\gamma_{ij} = B_{ij} V_i V_j$ for each edge connecting buses $i$ and $j$. We summarize the symbols used in Table 1. In this paper, we assume that the voltages at the buses are constant and the lines are lossless.