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Brief paper

Two-stage information filters for single and multiple sensors, and their square-root versions[☆]

Kumar Pakki Bharani Chandra [a,*], Mohamed Darouach [b]

[a] GMR Institute of Technology, Rajam-532127, Srikakulam (dt), Andhra Pradesh, India
[b] CRAN-CNRS (UMR 7039), Universite de Lorraine, IUT de Longwy, 186, Rue de Lorraine, 54400 Cosnes et Romain, France

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ABSTRACT

Accurate states and unknown random bias estimation for well- and ill-conditioned systems are crucial for several applications. In this paper, a fusion of a two-stage Kalman filter and an information filter, and its extensions are considered to estimate the state variables and unknown random bias. Specifically, we propose four extensions of two-stage Kalman filters: two-stage information filter (TSIF), multi-sensor two-stage information filter (M-TSIF) and their square-root versions. The TSIF deals with single-sensor systems whereas the M-TSIF is capable to handle multi-sensor systems. For ill-conditioned systems, numerically stable square-root versions of TSIF and M-TSIF are developed. The performance of the proposed filters (along with the existing two-stage Kalman filter), for well- and ill-conditioned cases, is demonstrated on a quadruple-tank model.

1. Introduction and preliminaries

1.1. Introduction

For many practical systems it is often difficult to obtain an accurate plant model due to various factors such as bias, un-modelled dynamics. State estimation of unknown random bias plays important role in various applications like fault-tolerant control design. The undesired effects of unknown random bias can be minimised or eliminated by designing an appropriate fault-tolerant controller based on the estimated bias. If the unknown bias estimate is not accurate, then there is a high chance that the overall fault-tolerant control system could fail to provide the desired performance. Several authors have explored various methods to estimate the unknown random bias using two-stage Kalman filters (Chen & Patton, 2012; Darouach & Zasadzinski, 1997; Darouach, Zasadzinski, & Boutayeb, 2003; Gillijns & De Moor, 2007; Hsieh & Chen, 1999; Ignagni, 1981; Keller & Darouach, 1997). A numerically stable square-root two stage Kalman filter is proposed in Kanev, Verhaegen, et al. (2005). In two-stage Kalman filters, two filters (bias-free estimator and bias estimator) are synthesised in parallel to estimate the bias. A square-root two-stage information filter and smoother for 'constant' bias is given in Bierman (1975). In certain real-life applications multi-sensor based estimation is preferred over a single-sensor based estimation, for example in target tracking applications, information from many radars placed at different altitudes are fused to reliably estimate the states of the target (Gan & Harris, 2001; Lin, Bar-Shalom, & Kirubarajan, 2004, 2005). Initialisation is easy in information filters than the covariance filters. However, unlike the Kalman filters, the state variables and covariance are not readily available for information filters; they have to be recovered at each time step for further processing. For multi-sensor state estimation, due to the simple update stage, the information filters are preferred over their algebraic equivalent Kalman filters (Grewal & Andrews, 2011; Mutambara, 1998). Most of the existing multi-sensor approaches for bias estimation assumes that only measurement models are affected by the bias; and the process or plant models are explicitly bias free (Lin et al., 2004, 2005). Further, they are developed to propagate the covariance matrices — which in some cases are not numerically reliable for ill-conditioned systems; for such systems square-root filters are advisable. The main contribution of this paper is to propose computationally efficient information forms of the two-stage Kalman filters to deal with measurements from multiple sensors, in the presence of process and measurement noise as well as random bias. In this paper, we first present a two-stage information filter (TSIF) and its square-root version for measurement from single sensor. Further we extend them to handle multiple sensors to estimate the state vector and the unknown random bias. The square-root versions of the proposed filters have been designed

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* Corresponding author.
E-mail addresses: bharanichandrakumar.p@gmrit.org (K.P. Bharani Chandra), mohamed.darouach@univ-lorraine.fr (M. Darouach).

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to handle ill-conditioned systems, where the square-root factors of information matrices are propagated. Further, in this paper, both plant and measurement models are assumed to be explicitly corrupted by bias.

1.2. Preliminaries

This section briefly presents the two-stage Kalman filter (Keller & Darouach, 1997).

Consider the following discrete-time process, observation and bias models

$$x_k = Ax_{k-1} + Bu_{k-1} + Fb_{k-1} + w^x_{k-1} \tag{1}$$

$$y_k = Cx_k + Gb_k + v_k \tag{2}$$

$$b_k = b_{k-1} + w^b_{k-1} \tag{3}$$

where, $x_k \in \mathcal{R}^n$ is the state vector, $u_k \in \mathcal{R}^r$ is the control input, $y_k \in \mathcal{R}^m$ is the measurement vector and $b_k \in \mathcal{R}^p$ is an unknown bias vector. w^x_k, v_k and w^b_k, are the process, observation and bias noises, which are assumed to be zero-mean uncorrelated random variables with Q, R and N as their respective covariances. The matrices A, B, F, C and G are of appropriate dimensions.

The two-stage Kalman filter consists of two parallel filters, namely bias-free estimator and bias estimator as given below.

The optimal two-stage Kalman state estimate for the system (1)–(3) is given by the following equation (Keller & Darouach, 1997):

$$x_{k|k} = \widetilde{x}_{k|k} + \beta_{k|k}\, b_{k|k} \tag{4}$$

and the following two algorithms.

Bias-free estimator:

1: The predicted state vector and covariance matrix are:

$$\begin{aligned}\widetilde{x}_{k|k-1} &= A\widetilde{x}_{k-1|k-1} + Bu_{k-1} + r_{k-1}b_{k-1|k-1}\\ &\quad -\beta_{k|k-1}b_{k-1|k-1}\end{aligned} \tag{5}$$

$$\begin{aligned}\widetilde{P}_{k|k-1} &= A\widetilde{P}_{k-1|k-1}A^T + Q + r_{k-1}P^b_{k-1|k-1}r^T_{k-1}\\ &\quad -\beta_{k|k-1}P^b_{k|k-1}\beta^T_{k|k-1}\end{aligned} \tag{6}$$

where,

$$r_{k-1} = A\beta_{k-1|k-1} + F \tag{7}$$

$$P^b_{k|k-1} = P^b_{k-1|k-1} + N \tag{8}$$

$$\beta_{k|k-1} = r_{k-1}P^b_{k-1|k-1}P^{b^{-1}}_{k|k-1} \tag{9}$$

2: The updated state vector and covariance matrix are:

$$\widetilde{x}_{k|k} = \widetilde{x}_{k|k-1} + \widetilde{K}_k\widetilde{v}_k \tag{10}$$

$$\widetilde{P}_{k|k} = (\mathbb{I} - \widetilde{K}_kC)\widetilde{P}_{k|k-1} \tag{11}$$

where, $\mathbb{I}$ is the identity matrix of an appropriate dimension and

$$\widetilde{K}_k = \widetilde{P}_{k|k-1}C^T\widetilde{G}^{-1}_k \tag{12}$$

$$\widetilde{G}_k = C\widetilde{P}^x_{k|k-1}C^T + R \tag{13}$$

$$\widetilde{v}_k = y_k - C\widetilde{x}_{k|k-1} \tag{14}$$

Bias estimator:

1: The predicted bias and corresponding covariance matrix are:

$$b_{k|k-1} = b_{k-1|k-1}, \quad P^b_{k|k-1} = P^b_{k-1|k-1} + N \tag{15}$$

2: The updated bias and corresponding covariance are:

$$b_{k|k} = b_{k|k-1} + K^b_k(\widetilde{v}_k - H_{k|k-1}b_{k|k-1}) \tag{16}$$

$$P^b_{k|k} = (\mathbb{I} - K^b_kH_{k|k-1})P^b_{k|k-1} \tag{17}$$

where,

$$K^b_k = P^b_{k|k-1}H^T_{k|k-1}\left(H_{k|k-1}P^b_{k|k-1}H^T_{k|k-1} + \widetilde{G}_k\right)^{-1} \tag{18}$$

$$H_{k|k-1} = G + C\beta_{k|k-1} \tag{19}$$

3: The updated β is given by:

$$\beta_{k|k} = \beta_{k|k-1} - \widetilde{K}_kH_{k|k-1} \tag{20}$$

2. Main results

2.1. Two-stage information filter and its square-root version

For many practical applications, information filters are preferred over their equivalent Kalman filters. Some of the preferred features of information filters include easy initialisation and computationally simple update stage, Kailath, Sayed, and Hassibi (2000), Lee (2008), Mutambara (1998), Psiaki (1999) and Wang, Feng, and Tse (2014).

2.1.1. Two-stage information filter

An equivalent information form of optimal two-stage Kalman filter given in Section 1.2 will be derived. In the information filter the inverse of the covariance matrix (information matrix) and information vector are propagated.

The information matrix and information vector can be written in terms of covariance matrix and state vector as

$$Y = P^{-1} = P \setminus \mathbb{I}, \quad y = Yx. \tag{21}$$

where, '\' is the *left division operator* (Chandra, Gu, & Postlethwaite, 2013; Eustice, Singh, Leonard, & Walter, 2006) and $\mathbb{I}$ denotes the identity matrix. Throughout this paper the above notation will be used. The proposed TSIF is given in the following theorem.

Theorem 1. *The optimal TSIF's bias and state vector estimate for a system in (1)–(3) are given by the following equations:*

$$x_{k|k} = \widetilde{Y}_{k|k}\backslash\widetilde{y}_{k|k} + \left[\widetilde{Y}_{k|k}\backslash y^\beta_{k|k}\right]\left[Y^b_{k|k}\backslash y^b_{k|k}\right] \tag{22}$$

$$b_{k|k} = \left[\widetilde{Y}^b_{k|k}\backslash y^b_{k|k}\right] \tag{23}$$

and the following algorithms.

Bias-free information filter:

1: *The predicted information vector and the corresponding information matrix are:*

$$\begin{aligned}\widetilde{Y}_{k|k-1} &= \left[A\widetilde{Y}_{k-1|k-1} \setminus \mathbb{I}A^T + Q + r_{k-1}Y^b_{k-1|k-1} \setminus \mathbb{I}r^T_{k-1}\right.\\ &\quad \left. -\beta_{k|k-1}\, Y^b_{k|k-1} \setminus \mathbb{I}\, \beta^T_{k|k-1}\right] \setminus \mathbb{I}\end{aligned} \tag{24}$$

$$\widetilde{y}_{k|k-1} = Y_{k|k-1}\widetilde{x}_{k|k-1}, \tag{25}$$

where,

$$\beta_{k|k-1} = r_{k-1}\left[Y^b_{k-1|k-1} \setminus \mathbb{I}\right]Y^b_{k|k-1}$$

$$\begin{aligned}\widetilde{x}_{k|k-1} &= A\widetilde{Y}_{k-1|k-1} \setminus \widetilde{y}_{k-1|k-1} + Bu_{k-1} + r_{k-1}b_{k-1|k-1}\\ &\quad -\beta_{k|k-1}b_{k-1|k-1}\end{aligned}$$

$$r_{k-1} = A\beta_{k-1|k-1} + F$$