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Brief paper

Optimal multirate sampling in symbolic models for incrementally stable switched systems[☆]

Adnane Saoud [a,b], Antoine Girard [a,*]

[a] Laboratoire des Signaux et Systèmes (L2S), CNRS, CentraleSupélec, Université Paris-Sud, Université Paris-Saclay 3, rue Joliot-Curie, 91192 Gif-sur-Yvette, France

[b] Laboratoire Spécification et Vérification, CNRS, ENS Paris-Saclay, 94235 Cachan Cedex, France

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ABSTRACT

Methods for computing approximately bisimilar symbolic models for incrementally stable switched systems are often based on discretization of time and space, where the value of time and space sampling parameters must be carefully chosen in order to achieve a desired precision. These approaches can result in symbolic models that have a very large number of transitions, especially when the time sampling, and thus the space sampling parameters are small. In this paper, we present an approach to the computation of symbolic models for switched systems with dwell-time constraints using multirate time sampling, where the period of symbolic transitions is a multiple of the control (i.e. switching) period. We show that all the multirate symbolic models, resulting from the proposed construction, are approximately bisimilar to the original incrementally stable switched system with the precision depending on the sampling parameters, and the sampling factor between transition and control periods. The main contribution of the paper is the explicit determination of the optimal sampling factor, which minimizes the number of transitions in the class of proposed symbolic models for a prescribed precision. Interestingly, we prove that this optimal sampling factor is mainly determined by the state space dimension and the number of modes of the switched system. Finally, an illustration of the proposed approach is shown on an example, which shows the benefit of multirate symbolic models in reducing the computational cost of abstraction-based controller synthesis.

1. Introduction

A switched system is a dynamical system consisting of a finite number of subsystems and a law that controls the switching among them (Liberzon, 2003; Lin & Antsaklis, 2009; Sun & Ge, 2011). The literature on switched systems principally focuses on the stability and stabilization problems. However, other objectives need also to be considered such as safety, reachability or more complex objectives such as those expressed in linear temporal logic. For this reason, over recent years, several studies focused on the use of discrete abstractions and symbolic control techniques. The area of symbolic control is concerned with the use of algorithmic discrete synthesis techniques for designing controllers for continuous and hybrid dynamical systems (see e.g. Rungger, Girard, & Tabuada, 2017; Tabuada, 2009 and the references therein). A key concept in symbolic control is that of symbolic models, which consist in discrete abstractions of the continuous dynamics, and which are amenable to automata theoretic techniques for the synthesis of controllers enforcing a broad range of specifications (Belta, Yordanov, & Aydin Gol, 2017; Bloem, Jobstmann, Piterman, Pnueli, & Sa'ar, 2012). Controllers for the original system, with strong formal guarantees, can then be obtained through dedicated refinement procedures (Girard, 2012; Reissig, Weber, & Rungger, 2017; Tabuada, 2009). This latter step requires the original system and the symbolic model to be related by some formal behavioral relationship such as simulation, bisimulation relations or their approximate and alternated versions (Girard & Pappas, 2007; Tabuada, 2009).

Numerous works have been dedicated to the computation of symbolic models for various classes of dynamical systems. Focusing on approximately bisimilar abstractions, existing approaches

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* Corresponding author.
E-mail addresses: Adnane.Saoud@l2s.centralesupelec.fr (A. Saoud), Antoine.Girard@l2s.centralesupelec.fr (A. Girard).

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make it possible to deal with nonlinear systems (Pola, Girard, & Tabuada, 2008; Pola & Tabuada, 2009), switched systems (Girard, Pola, & Tabuada, 2010), time-delay systems (Pola, Pepe, & Di Benedetto, 2015; Pola, Pepe, Di Benedetto, & Tabuada, 2010), networked control systems (Borri, Pola, & Di Benedetto, 2014; Zamani, Mazo, & Abate, 2014), stochastic systems (Zamani & Abate, 2014; Zamani, Esfahani, Majumdar, Abate, & Lygeros, 2014), etc. All these approaches are essentially based on discretization of time and space and require the considered system to satisfy some kind of incremental stability property (Angeli, 2002). However, incremental stability can be dropped if one seeks symbolic models related only by one-sided approximate simulation relations (Tabuada, 2008; Zamani, Pola, Mazo, & Tabuada, 2012). In most cases, symbolic models of arbitrary precision can be obtained by carefully choosing time and space sampling parameters. However, for a given precision, the choice of a small time sampling parameter imposes to choose a small space sampling parameter resulting in symbolic models with a prohibitively large number of transitions. This constitutes a limiting factor of the approach because the size of the symbolic models is crucial for computational efficiency of discrete controller synthesis algorithms. Several studies have been conducted in order to address this issue by enabling the computation of more parsimonious symbolic models with smaller numbers of transitions. Such approaches include compositional abstraction schemes where symbolic models of a system are built from symbolic models of its components (Majumdar, Mallik, & Schmuck, 2016; Pola, Pepe, & Di Benedetto, 2016; Tazaki & Imura, 2008); multi-resolution or multi-scale symbolic models computed using non-uniform adaptive space discretizations (Girard, Gössler, & Mouelhi, 2016; Tazaki & Imura, 2009); symbolic models where the set of symbolic states is not given by a discretization of the state-space but by input sequences (Le Corronc, Girard, & Gössler, 2013; Zamani, Abate, & Girard, 2015).

In this paper, we show how the size of symbolic models can be reduced using multirate sampling. Multirate sampling has been introduced in the area of sampled-data systems to face some of the sampling processes disadvantages such as the loss of relative degree and changes in the properties of the zero dynamics (see e.g. Grizzle & Kokotovic, 1988; Monaco & Normand-Cyrot, 1992, 2001). In this paper, we present an approach to the computation of multirate symbolic models for incrementally stable switched systems, where the period of symbolic transitions is a multiple of the control (i.e. switching) period. A similar approach has been explored in the symbolic control literature in the context of nonlinear digital control systems (Majumdar & Zamani, 2012). The first contribution of the paper is to extend this approach to the class of switched systems, with dwell-time constraints. We show that the obtained multirate symbolic models are approximately bisimilar to the original switched system. Then, the second and main contribution of the paper lies in the explicit determination of the optimal sampling factor between transition and control periods, which minimizes the number of transitions in the class of proposed symbolic models for a prescribed precision; this problem is not considered in Majumdar and Zamani (2012). Interestingly, we show that the optimal sampling factor is mainly determined by the state space dimension and the number of modes of the switched system.

This paper is organized as follows. In Section 2, we introduce the class of incrementally stable switched systems under study and we present the abstraction framework used in the paper. In Section 3, we present the construction of symbolic models for incrementally stable switched systems with dwell-time constraints, using multirate sampling. In Section 4, we establish the optimal sampling factor between control and transition periods which minimizes the number of transitions in the symbolic model. Finally, in Section 5, we illustrate our approach using an example taken from Girard et al. (2010), which shows the benefits of the proposed multirate symbolic models.

A preliminary version of this work has been presented in the conference paper (Saoud & Girard, 2017) where switched systems without dwell-time constraints are considered. The current paper extends the approach to consider dwell-time constraints; results of Saoud and Girard (2017) being recovered as particular cases. We also provide novel numerical experiments.

Notations. $\mathbb{Z}, \mathbb{N}$ and $\mathbb{N}^+$ denote the sets of integers, of non-negative integers and of positive integers, respectively. $\mathbb{R}, \mathbb{R}_0^+$ and $\mathbb{R}^+$ denote the sets of real numbers, of non-negative real numbers, and of positive real numbers, respectively. For $s \in \mathbb{R}_0^+$, $\lfloor s \rfloor$ denote its integer part, i.e. the largest nonnegative integer $r \in \mathbb{N}$ such that $r \leq s$. For $x \in \mathbb{R}^n$, $\|x\|$ denotes the Euclidean norm (i.e. the 2-norm) of x. A continuous function $\gamma : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\gamma(0) = 0$; γ is said to belong to class $\mathcal{K}_\infty$ if γ is of class $\mathcal{K}$ and $\gamma(s) \to \infty$ as $s \to \infty$. A continuous function $\beta : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said to belong to class $\mathcal{KL}$ if, for all fixed $t \in \mathbb{R}_0^+$, the map $\beta(\cdot, t)$ belongs to class $\mathcal{K}$, and for all fixed $s \in \mathbb{R}^+$, the map $\beta(s, \cdot)$ is strictly decreasing and $\beta(s, t) \to 0$ as $t \to \infty$.

2. Preliminaries

2.1. Incrementally stable switched systems

We introduce the class of switched systems:

Definition 1. A switched system is a quadruple $\Sigma = (\mathbb{R}^n, P, \mathcal{P}, F)$, consisting of the following:

- a state space $\mathbb{R}^n$;
- a finite set of modes $P = \{1, \ldots, m\}$;
- a set of switching signals $\mathcal{P} \subseteq \mathcal{S}(\mathbb{R}_0^+, P)$, where $\mathcal{S}(\mathbb{R}_0^+, P)$ denotes the set of piecewise constant functions from $\mathbb{R}_0^+$ to P, continuous from the right and with a finite number of discontinuities on every bounded interval of $\mathbb{R}_0^+$;
- a collection of vector fields $F = \{f_1, \ldots, f_m\}$, indexed by P.

The discontinuities $0 < t_1 < t_2 < \ldots$ of a switching signal are called *switching times*; by definition of $\mathcal{S}(\mathbb{R}_0^+, P)$, there are only a finite number of switching times on every bounded interval of $\mathbb{R}_0^+$ and thus Zeno behaviors are avoided. A switching signal $\mathbf{p} \in \mathcal{S}(\mathbb{R}_0^+, P)$ has *dwell-time* $\tau_d \in \mathbb{R}^+$ if the sequence of switching times satisfies $t_{k+1} - t_k \geq \tau_d$, for all $k \geq 1$. The set of switching signals with dwell-time τ_d is denoted $\mathcal{S}_{\tau_d}(\mathbb{R}_0^+, P)$.

A piecewise $\mathcal{C}^1$ function $\mathbf{x} : \mathbb{R}_0^+ \to \mathbb{R}^n$ is said to be a *trajectory* of Σ if it is continuous and there exists a switching signal $\mathbf{p} \in \mathcal{P}$ such that, at each $t \in \mathbb{R}_0^+$ where the function $\mathbf{p}$ is continuous, $\mathbf{x}$ is continuously differentiable and satisfies

$$\dot{\mathbf{x}}(t) = f_{\mathbf{p}(t)}(\mathbf{x}(t)). \quad (1)$$

We make the assumption that the vector fields $f_p, p \in P$, are locally Lipschitz and forward complete (see e.g. Angeli & Sontag, 1999 for necessary and sufficient conditions), so that for all switching signals $\mathbf{p} \in \mathcal{P}$ and all initial states $x \in \mathbb{R}^n$, there exists a unique trajectory, solution to (1) with $\mathbf{x}(0) = x$, denoted $\mathbf{x}(., x, \mathbf{p})$. We will denote by ϕ_t^p the flow associated to the vector field f_p. Then, for a constant switching signal given by $\mathbf{p}(t) = p$, for all $t \in \mathbb{R}_0^+$, we have $\mathbf{x}(t, x, \mathbf{p}) = \phi_t^p(x)$, for all $t \in \mathbb{R}_0^+$.

In the following, we consider *incrementally globally uniformly asymptotically stable* (δ-GUAS) switched systems, see Girard et al. (2010) for a formal definition. Intuitively, incremental stability means that all trajectories associated to the same switching signal converge to the same trajectory, independently of their initial conditions. Sufficient conditions for incremental stability are given in Girard et al. (2010) in terms of existence of multiple Lyapunov functions.