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Brief paper

Suboptimal receding horizon estimation via noise blocking☆

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ABSTRACT

For discrete-time linear systems, we propose a suboptimal approach to constrained estimation so that the associated computation burden is reduced. This is achieved by enforcing a move blocking (MB) structure in the estimated process noise sequence (PNS). We show that full information estimation (FIE) and receding horizon estimation (RHE) with MB are both stable in the sense of an observer. The techniques in proving stability are inspired by those that have been proposed for standard RHE. To be specific, stability results are mainly achieved by (i) carefully embellishing the general assumptions for standard RHE to accommodate the MB requirement; (ii) exploiting the principle of optimality, as well as convexity of the quadratic programs (QPs) associated with FIE and RHE; (iii) relying on the fact that the Kalman filter is the best linear estimator in the least-squares sense. A crucial requirement in achieving stability for MB RHE is that the segment structure (SS) of the PNS of MB FIE for the optimization steps within the receding horizon (i.e., steps between $T-N$ and $T-1$) has to be enforced in the MB RHE optimization. As a result, the MB RHE strategy becomes a *dynamic* estimator with a periodically varying computational complexity. The theoretical results have been illustrated with examples.

1. Introduction

Including constraints in estimation can lead to improved performance in applications (Dissanayake, Sukkarieh, Nebot, & Durrant-Whyte, 2001; Gorinevsky, Kim, Beard, Boyd, & Gordon, 2009; Mahata, Mousavi, Söderström, & Valdek, 2005; Xie, Ugrinovskii, & Petersen, 2008; Yan & Bitmead, 2005). RHE, as the counterpart of receding horizon control (RHC) (Kong, Goodwin & Seron, 2013; Goodwin, Kong, Mirzaeva & Seron, 2014) for estimation, is a systematic framework for handling constraints (Goodwin, De Dona, Seron, & Zhuo, 2005; Goodwin, Seron, & Doná, 2005; Rawlings & Ji, 2012; Rawlings & Mayne, 2009; Samar, Gorinevsky, & Boyd, 2004). In contrast to FIE that uses all available information (see, e.g., Ge & Kerrigan, 2017, Muske, Rawlings, & Lee, 1993), RHE only employs measurements within a receding time frame, and the information embedded in the previous measurements is captured by the arrival cost (Rawlings & Mayne, 2009, pp. 32-40). Various forms of RHE have been proposed so far. In Goodwin, De Dona et al. (2005), Goodwin, Seron et al. (2005), Kong and Sukkarieh (2018), Muske et al. (1993), Rao (2000), Rawlings and Mayne (2009), Rawlings and Ji (2012), Rao, Rawlings, and Lee (2001), Rao, Rawlings, and Mayne (2003) and Samar et al. (2004), the data fitting cost is optimized over the initial state and the PNS. Other frameworks estimate only the initial state (Alessandri, Baglietto, & Battistelli, 2003; Michalska & Mayne, 1995; Sui & Johansen, 2014). Moreover, some useful techniques have been proposed in the recent literature to combine RHE with particle filtering (Rawlings & Bakshi, 2006) or linear estimation methods (Kong & Sukkarieh, 2018; Sui & Johansen, 2014).

Despite that FIE and RHE can offer appealing performance, one has to solve an optimization problem at each sampling time. This requirement can be alleviated in a few different ways. For example, in close spirit to the well-known results on explicit RHC (Bemporad, Morari, Dua, & Pistikopoulos, 2002; Johansen, Petersen, & Slupphaug, 2002; Kerrigan & Maciejowski, 2004), Voelker, Kouramas, and Pistikopoulos (2013) formulate the RHE as a multi-parametric QP and show that the optimal solution is a piecewise affine function of the measurements, the known inputs, and the arrival cost. One can also explore the optimization structure to accelerate the computation (Gorinevsky et al., 2009; Morabito, Kogel, Bullinger, Pannocchia, & Findeisen, 2015). Whilst it is imperative to render fast solutions, it is equally important that the estimation error stability is guaranteed (Alessandri & Gaggero, 2017; Schneider, Hannemann-Tamás, & Marquardt, 2015). Especially, in Alessandri and Gaggero (2017), descent algorithms have been proposed to solve RHE for linear and nonlinear systems. Conditions that ensure the estimation error stability are also thoroughly discussed

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therein. Schneider et al. (2015) deal with large-scale linear systems, and rigorously derive conditions guaranteeing convergence to the optimal centralized RHE so that stability properties of the latter hold for the considered framework.

Alternative to the above methods, this paper proposes to reduce the computation burden of constrained estimation by enforcing a MB structure on the estimated PNS in the optimization. Note that the MB framework and the methods in Alessandri and Gaggero (2017) and Schneider et al. (2015) have their respective (and complementary) features. To be specific, within our RHE framework (see in Section 2), the cost is optimized over both the initial state and the PNS, accounting for constraints. On the contrary, Alessandri and Gaggero (2017) consider disturbance-free systems, and try to estimate only the initial state without considering constraints. An advantage of the results in Alessandri and Gaggero (2017) is that they are applicable for both linear and nonlinear systems, while we only consider linear systems, although the extension of the MB concept to nonlinear systems is possible, based on the ideas herein and Rao et al. (2003). Our results are similar with those in Schneider et al. (2015) in that both frameworks estimate the initial state and the PNS while considering constraints, for linear systems. However, we consider the centralized case for both FIE and RHE while Schneider et al. (2015) focus on distributed RHE.

Our contributions are summarized as follows. (1) Since the complexity of FIE and RHE with MB depends on block size (taken to be fixed, say S steps) w.r.t. the sampling instant order (in time) and the receding horizon length, we give a thorough analysis and illustration on the segment structures (SS) of the PNS for FIE and RHE (see in Figs. 1–4). (2) A second major contribution is to show that both FIE and RHE with MB are stable in the sense of an observer. The techniques we adopt are inspired by those given in Rao et al. (2001) for standard RHE. Stability is mainly achieved by (i) carefully embellishing standard assumptions in Rao et al. (2001) to accommodate the MB requirement (see, Assumption **A1**); (ii) exploiting the principle of optimality, and convexity of the QPs associated with FIE and RHE (Proposition 1); (iii) relying on the fact that Kalman filter (KF) is the best linear estimator in the least-squares sense without constraints (Remark 2 and Lemma 2). (3) We have discovered a crucial requirement in proving stability for RHE with MB, i.e., given time T and the horizon length N, the SS of the PNS of FIE with MB for the time steps between $T-N$ and $T-1$, has to be enforced for RHE. (4) We have illustrated the proposed strategy with several simulation studies.

Ideas of input parametrization and MB have been proposed in RHC (Maciejowski, 2002, pp. 159–163), (Wang, 2001, 2004, 2009, chps. 3–6), (Cagienard, Grieder, Kerrigan, & Morari, 2007; Gondhalekar & Imura, 2010; Gondhalekar, Imura, & Kashima, 2009; Goodwin et al., 2006; Jung, Jang, & Lee, 2015; Li, Xi, & Lin, 2013; Shekhar & Maciejowski, 2012). Especially, recent research in Wang (2001), Wang (2004) and Wang (2009) shows that basis functions such as Laguerre and Kautz functions can be utilized in parametrizing RHC so that the number of optimization variables is greatly reduced with stability guarantee. Our conjecture is that similar ideas can also be applied in RHE, resulting in comparable (or better) computation and estimation tradeoffs with these of the MB method, especially for noises having large temporal variability. However, a complete comparison of the basis function and MB concepts in RHE is out of this paper's scope, and thus left for future work. We will also show that RHE with MB has similarities and differences with MB RHC. On the one hand, the complexity of the associated optimization problem for RHE with MB turns out to be changing periodically, given the requirement mentioned in item (3) in the previous paragraph (see in (11)–(12)). This is similar to the finding of Cagienard et al. (2007) in the sense that RHC with MB is no longer a static, but a dynamic controller.

On the other hand, the techniques we adopt in proving stability differ drastically from those in RHC with MB. Since MB can destroy recursive feasibility in RHC, the main issue in establishing stability is how to maintain recursive feasibility so that the optimal cost is decreasing and can be used as a Lyapunov function as in standard RHC (Cagienard et al., 2007; Gondhalekar & Imura, 2010; Gondhalekar et al., 2009; Li et al., 2013; Shekhar & Maciejowski, 2012). In constrained estimation with MB, however, the optimal cost is *not* monotonically decreasing. For FIE with MB, the optimal cost proves to be monotonically nondecreasing with time (Proposition 1, Corollary 1); for RHE with MB, the optimal cost is monotonically nondecreasing for each N steps in time (Proposition 2 and Remark 3). These statements hold regardless of the MB size w.r.t. the receding horizon length, and are key to obtaining stability (Theorems 1–2).

The reminder of the paper is structured as follows. Section 2 recalls preliminaries on FIE and RHE, and proposes the idea of MB in constrained estimation. Section 3 considers FIE with MB and presents a detailed analysis of the SS of the PNS and stability results. Section 4 discusses RHE with MB and possible extensions. In Section 5, several numerical simulation results are presented. Section 6 concludes the paper.

Notation: $[a_1, \ldots, a_n]$ denotes $[a_1^{\mathrm{T}} \cdots a_n^{\mathrm{T}}]^{\mathrm{T}}$, where $a_1, \ldots, a_n$ are scalars/vectors/matrices of proper dimensions. The weighted Euclidean norm is denoted by $\|x\|_{R^{-1}}^2 = x^{\mathrm{T}} R^{-1} x$. $\mathbb{Z}$ and $\mathbb{Z}^+$ denote the sets of non-negative and positive integers, respectively. $\mathscr{I}_i^j$ denotes the set of integers between i and j. $\mathcal{OS}_{t_1}^{t_2}$ denotes the optimization steps (OS) between t_1 and t_2 for a given optimization problem. $\mathbf{1}_n$ denotes a column vector of n dimension having 1 as its elements. Given $\boldsymbol{d} = [d_0, d_1, \ldots]$, $\mathbf{d}_\tau^\mu = [d_\tau, \ldots, d_\mu]$ denotes the truncation of $\mathbf{d}$ between the τth and the μth index. $\otimes$ denotes the Kronecker product operation.

2. Preliminaries and problem formulation

Consider the discrete-time linear time-invariant system

$$x_{k+1} = Ax_k + Gw_k, \ \ y_k = Cx_k + v_k \tag{1}$$

where, the state, the process and measurement noise vectors satisfy $x_k \in \mathcal{X} \subset \mathbf{R}^n$, $w_k \in \mathcal{W} \subset \mathbf{R}^m$ and $v_k \in \mathcal{V} \subset \mathbf{R}^q$, respectively; (A, C) is observable; the sets $\mathcal{X}, \mathcal{W}$, and $\mathcal{V}$ are compact and convex with $\mathcal{W}$ and $\mathcal{V}$ containing the origin in their interior. Denote $x(k; x_s, \mathbf{w}_0^{k-1}) = A^k x_s + \sum_{i=0}^{k-1} A^{k-i-1} G w_i$ as the solution of system (1) at the kth sampling time, with the initial state x_s and the noise sequence $\mathbf{w}_0^{k-1} = [w_0, w_1 \cdots, w_{k-1}]$. Variables (x_k, w_k, y_k, v_k) in (1) refer to the *real* process. They have associated decision and optimal decision variables in optimization, which we denote as $(\chi_k, \omega_k, \eta_k, \upsilon_k)$ and $(\widehat{x}_k, \widehat{w}_k, \widehat{y}_k, \widehat{v}_k)$, respectively, and these satisfy

$$\chi_{k+1} = A\chi_k + G\omega_k, y_k = C\chi_k + \upsilon_k, \eta_k = C\chi_k,$$
$$\widehat{x}_{k+1} = A\widehat{x}_k + G\widehat{w}_k, y_k = C\widehat{x}_k + \widehat{v}_k, \widehat{y}_k = C\widehat{x}_k,$$

where, υ_k and $\widehat{v}_k$ stand for the fitting error and the optimal fitting error, respectively. Denote $\omega_0^{T-1} = [\omega_0, \omega_0 \cdots, \omega_{T-1}]$, $\chi_k = \chi(k; \chi_0, \omega_0^{k-1})$, $\upsilon_k = y_k - C\chi_k$. The standard FIE in a *prediction* form is:

$$\mathscr{F}_T : \begin{cases} \min\limits_{\chi_0, \omega_0^{T-1}} \phi_T \text{ s.t. } \chi_k \in \mathcal{X}, k \in \mathscr{I}_0^T \\ \omega_k \in \mathcal{W}, \upsilon_k \in \mathcal{V}, \ k \in \mathscr{I}_0^{T-1} \end{cases}, \tag{2}$$

where, $\phi_T = \left\| \chi_0 - x_g \right\|_{\Pi_0^{-1}}^2 + \sum_{k=0}^{T-1} \left[\|\upsilon_k\|_{R^{-1}}^2 + \|\omega_k\|_{Q^{-1}}^2 \right]$, Π_0, R, $Q > 0$. In ϕ_T, (x_g, Π_0) summarizes the *prior* information at time