Automatica 98 (2018) 247–255

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Adaptive output feedback control of stochastic nonholonomic systems with nonlinear parameterization[☆]

Hui Wang[a,b,c], Quanxin Zhu[a,b,*]

[a] Key Laboratory of HPC-SIP (MOE), College of Mathematics and Statistics, Hunan Normal University, Changsha, 410081, China
[b] School of Mathematical Sciences and Institute of Finance and Statistics, Nanjing Normal University, Nanjing, 210023, China
[c] School of Mathematics and Statistics, Nanjing University of Information Science and Technology, Nanjing, 210044, China

ARTICLE INFO

Article history:
Received 18 September 2017
Received in revised form 7 April 2018
Accepted 11 August 2018

Keywords:
Stochastic nonholonomic system
Globally asymptotic stability
Nonlinear parameterization
Adaptive control
Output-feedback

ABSTRACT

The problem of adaptive output-feedback control of nonlinearly parameterized stochastic nonholonomic systems is studied in this paper. Since many unknowns (e.g., unknown control coefficients and unknown nonlinear parameters in systems' nonlinearities) occur into systems, we utilize an adaptive control method, together with a parameter separation technique, to construct an adaptive output feedback controller to regulate the whole systems. During the design procedure, a new form of reduced-order K-filters is given to compensate the unmeasured states of the systems. A switching strategy is proposed explicitly to stabilize the entire systems in the control scheme. Finally, a bilinear model with stochastic disturbances is presented to demonstrate our theoretical results.

1. Introduction

Nonholonomic systems, which are widely used in engineering models, such as wheeled mobile robot, knife edge and rolling disk, have received a great deal of attention over the past few decades. Undoubtedly, the stability analysis is one of hot issues in the research. However, Brockett in Brockett (1983) claimed that nonholonomic systems cannot be stabilized by static continuous state-feedback control laws though they are controllable. To overcome the drawback, three skillful approaches have been proposed, i.e., discontinuous time-invariant stabilization (Astolfi, 1996; Jiang, 2000), smooth time-varying stabilization (Tian & Li, 2002), and hybrid stabilization (Pomet, Thuilot, Bastin, & Campion, 1992). Since then, numerous related results have been achieved. For example, see Defoort, Demesure, Zuo, Polyakov, and Djemai (2016), Do (2015), Du, Wang, Wang, and Zhang (2015), Gao, Yuan, and Wu (2012), Hong, Wang, and Xi (2005), Lin, Pongvuthithum, and Qian (2002), Wang, Ge, and Lee (2004), Wu, Gao, and Liu (2015), Xi, Feng, Jiang, and Cheng (2007), Yuan and Qu (2010) and references therein.

However, when the systems' nonlinear terms allow unknown parameters, the traditional control methods used in the aforementioned works are invalid since there is not a priori knowledge of uncertainties in systems' nonlinearities. As unknown parameters occur into systems, adaptive control of such systems has been paid more attention by researchers. In Lin and Pongvuthithum (2000), Lin and Pongvuthithum proposed an adaptive design method, together with adding a power integrator technique, to regulate a class of high-order nonholonomic systems in chained form. Subsequently, Zhang & Liu (2012) applied this method to solve the adaptive state-feedback control of high-order nonholonomic systems with drift terms. Zhao, Yu, and Wu (2011) successfully extended the results to the stochastic case. In Ge, Wang, and Lee (2003), Ge et al. adopted the adaptive control method to design an output-feedback control law for a class of uncertain nonholonomic systems with strong nonlinear drifts. More results can be found in Liu and Zhang (2005), Zheng and Wu (2009) and references therein.

Note that the above works mainly consider the problem of adaptive control for linearly parameterized nonholonomic systems. As shown in Lin and Qian (2002), nonlinear parameterization is very common in many practical control problems. For nonlinearly parameterized ones, Lin & Qian (2002) proposed a useful tool—*parameter separation technique* to solve the adaptive control problem of nonlinearly parameterized systems. Based on this idea and stochastic stability theory in Khasminskii (1980), Xie and Tian (2009) designed an adaptive state-feedback controller to regulate a class of high-order stochastic nonlinear systems in lower-triangular form with nonlinear parameterization.

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Changyun Wen under the direction of Editor Miroslav Krstic.

* Corresponding author at: Key Laboratory of HPC-SIP (MOE), College of Mathematics and Statistics, Hunan Normal University, Changsha, 410081, China.
E-mail addresses: wang_8063085@163.com (H. Wang), zqx22@126.com (Q.X. Zhu).

https://doi.org/10.1016/j.automatica.2018.09.026
0005-1098/

Later, by means of the parameter separation technique, the authors in Gao and Yuan (2013) and Gao, Yuan, and Wu (2015) investigated the adaptive state-feedback control of stochastic nonholonomic systems with nonlinear parameterization. Naturally, an interesting unsolved question arose: *Can we achieve the adaptive output-feedback stabilization of nonlinearly parameterized stochastic nonholonomic systems?*

In this paper, we will attempt to answer the question. The main contributions of this paper are concluded as follows:

(i) The problem of adaptive control for stochastic nonholonomic systems is studied in this paper. Compared with the deterministic cases considered in Ge et al. (2003), Lin and Pongvuthithum (2000) and Liu and Zhang (2005) and Zheng and Wu (2009), the control design of such systems is an extremely challenging work due to the appearance of unknowns and stochastic disturbances. In addition, since nonlinear parameters occur into systems' nonlinearities, the conditions imposed on the systems will be less conservative. Based on the adaptive control method, together with a parameter separation technique, a switching output-feedback controller with an update law is developed recursively to guarantee that the entire system is globally asymptotically stable in probability at the origin.

(ii) Since the systems considered in this paper admit unknown control coefficients, the traditional observers proposed in Ge et al. (2003), Liu and Zhang (2005), Xi et al. (2007) and Zheng and Wu (2009) do not work here. Therefore, a new form of reduced-order K-filters is introduced into our control scheme to compensate the unmeasured states. A primary advantage of the proposed K-filters is that the computation of the matrix P used in the control scheme (see (9)) can be simplified greatly. Finally, a bilinear model with stochastic disturbances is presented to show the effectiveness of our results.

The rest of this paper is organized as follows. Section 2 presents some preliminary lemmas and assumptions. In Section 3, a switching control strategy, together with a new form of reduced-order K-filters, is presented explicitly. An adaptive output-feedback controller with the corresponding adaption law is designed step by step to regulate the whole system. Section 4 gives a simulation example to illustrate our theoretical results. Finally, we conclude this paper with some general remarks.

2. Preliminaries and assumptions

Note: Throughout this paper, $\mathbb{R}_+$ denotes the set of positive real numbers. $\mathbb{R}^n$ denotes the Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrix. $\text{sgn}(x)$ denotes the sign of the real number x. For any vector $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm of vector x. For any matrix $X \in \mathbb{R}^{n\times m}$, $\|X\|$ denotes the corresponding norm induced by the Euclidean norm of its vector. $A = (a_{ij})_{N\times N}$ denotes a matrix of N-dimension, A^T denotes the transpose of A, $\text{tr}\{A\}$ denotes its trace and $\|A\|_F = \sqrt{\text{tr}(A^TA)}$, $C = \text{diag}(c_1, c_2, \ldots, c_n)$ means C is a diagonal matrix, I_n means the n-dimensional identity matrix.

Let $w = (w_1(t), \ldots, w_r(t))^T$ be an r-dimensional Brownian motion defined in a complete probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. In this paper, we consider the following stochastic nonholonomic system with nonlinear parameterization

$$\begin{cases} dz = (du_0 + c_0 z)dt, \\ d\zeta_i = (d_i\zeta_{i+1}u_0 + \bar{f}_i(z,\zeta,\theta))dt + \bar{g}_i^T(z,\zeta,\theta)dw, \\ d\zeta_n = (d_n u_1 + \bar{f}_n(z,\zeta,\theta))dt + \bar{g}_n^T(z,\zeta,\theta)dw, \\ y = \zeta_1, \quad i = 1,\ldots,n-1, \end{cases} \tag{1}$$

where $(z, \zeta^T)^T = (z, \zeta_1, \ldots, \zeta_n)^T \in \mathbb{R}^{n+1}$ is the system state with the initial data $(z(t_0), \zeta^T(t_0))^T \in \mathbb{R}^{n+1}$, where t_0 is the initial time; $u_0, u_1 \in \mathbb{R}$ are system's control inputs; d_i, $i = 1, \ldots, n$, are unknown control coefficients; d is a known nonzero constant; $\theta \in \mathbb{R}$ is an unknown bounded constant parameter; c_0 is a known constant; functions $\bar{f}_i : \mathbb{R}\times\mathbb{R}^n\times\mathbb{R} \to \mathbb{R}$ and $\bar{g}_i : \mathbb{R}\times\mathbb{R}^n\times\mathbb{R} \to \mathbb{R}^r$, $i = 1, \ldots, n$, are locally Lipschitz in z and ζ satisfying $\bar{f}_i(0, 0, \theta) = 0$, $\bar{g}_i(0, 0, \theta) = 0$. The aim of this paper is to design an adaptive output-feedback controller

$$u_0 = u_0(z), \quad u_1 = u_1(z, \zeta_1, \vartheta), \quad \dot{\vartheta} = \nu(z, \zeta_1, \vartheta),$$

such that system (1) is globally asymptotically stable in probability at the origin.

In what follows, we will give some useful lemmas. Consider the following n-dimensional stochastic parameterized nonlinear system

$$dx = f(x,\theta)dt + g(x,\theta)dw, \quad x(t_0) = x_0 \in \mathbb{R}^n, \tag{2}$$

where $t \geq t_0$, functions $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^{n\times r}$ are locally Lipschitz in x with $f(0,\theta) = 0$ and $g(0,\theta) = 0$.

Let $\mathcal{C}^2(\mathbb{R}^n; \mathbb{R}_+)$ be the family of all nonnegative functions V on $\mathbb{R}^n$ which are continuously twice differentiable in x. For each $V \in \mathcal{C}^2(\mathbb{R}^n; \mathbb{R}_+)$, define an operator $\mathcal{L}V$ associated with (2) as follows:

$$\mathcal{L}V = \frac{\partial V}{\partial x^T}f + \frac{1}{2}\text{tr}\left\{g^T\frac{\partial^2 V}{\partial x^2}g\right\}. \tag{3}$$

Lemma 1 (*Gao & Yuan, 2013; Krstic & Deng, 1998*). *For system (2), suppose that there exists a function $V \in \mathcal{C}^2(\mathbb{R}^n; \mathbb{R}_+)$, class $\mathcal{K}_\infty$ functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ and a class $\mathcal{K}$ function $\alpha_3(\cdot)$ such that for all $x \in \mathbb{R}^n$, $t \geq t_0$*

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|), \quad \mathcal{L}V \leq -\alpha_3(|x|),$$

then the following statements hold: (i) there exists a unique solution on $[t_0, \infty)$; (ii) the solution $x = 0$ of the system (2) is globally asymptotically stable in probability (GAS-P).

Lemma 2 (*Xie & Tian, 2009*). *Suppose that c and d are two positive real numbers. Then, for any positive function $\gamma : \mathbb{R}\times\mathbb{R} \to \mathbb{R}_+$, we have*

$$|x|^c|y|^d \leq \frac{c}{c+d}\gamma(x,y)|x|^{c+d} + \frac{d}{c+d}(\gamma(x,y))^{-\frac{c}{d}}|y|^{c+d}.$$

Lemma 3 (*Lin & Qian, 2002*). *(Parameter Separation Technique) For any continuous function $f : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}$, there are smooth scalar functions $a(x) \geq 1$ and $b(y) \geq 1$, such that $|f(x,y)| \leq a(x)b(y)$.*

To proceed our procedure, we need the following assumptions imposed on system (1):

Assumption 1. For $\bar{f}_i$ and $\bar{g}_i$ ($i = 1, \ldots, n$), there exist known nonnegative smooth functions $\psi_i(\cdot)$ and $\phi_i(\cdot)$ such that for all $t \geq t_0, z \in \mathbb{R}, \zeta \in \mathbb{R}^n$,

$$|\bar{f}_i(z,\zeta,\theta)| \leq |y|\psi_i(z,y,\theta), \quad |\bar{g}_i(z,\zeta,\theta)| \leq |y|\phi_i(z,y,\theta).$$

Remark 1. The adaptive state-feedback control of system (1) has been fully addressed in Du, Wang, and Wang (2015) and Gao and Yuan (2013). However, the problem of adaptive output-feedback control for such a system has not yet been solved. Since the states $\zeta_2, \ldots, \zeta_n$ in system (1) are unmeasurable, these states cannot be applied in the control scheme. Different from the traditional observers used in Ge et al. (2003), Liu and Zhang (2005), Xi et al. (2007) and Zheng and Wu (2009), we propose a new form of reduced-order K-filters (see (8) below) to compensate the unmeasured states of system (1). Besides, Assumption 1 implies that system (1) admits nonlinear parameters. Based on Lemma 3, we know that there exist smooth functions $\bar{\psi}_i(\cdot) \geq 1$, $\bar{\phi}_i(\cdot) \geq 1$, $c_{f,i}(\cdot) \geq 1$ and $c_{g,i}(\cdot) \geq 1$ satisfying

$$\begin{aligned} &|\bar{f}_i(z,\zeta,\theta)| \leq |y|\bar{\psi}_i(z,y)c_{f,i}(\theta) \leq |y|\bar{\psi}_i(z,y)\Theta, \\ &|\bar{g}_i(z,\zeta,\theta)| \leq |y|\bar{\phi}_i(z,y)c_{g,i}(\theta) \leq |y|\bar{\phi}_i(z,y)\Theta. \end{aligned} \tag{4}$$