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Permanent magnet synchronous motors are globally asymptotically stabilizable with PI current control[☆]

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ABSTRACT

This note shows that the industry standard desired equilibrium for permanent magnet synchronous motors (*i.e*, maximum torque per Ampere) can be globally asymptotically stabilized with a PI control around the current errors, provided some viscous friction (possibly small) is present in the rotor dynamics and the proportional gain of the PI is suitably chosen. Instrumental to establish this surprising result is the proof that the map from voltages to currents of the incremental model of the motor satisfies some passivity properties. The analysis relies on basic Lyapunov theory making the result available to a wide audience.

1. Introduction

Control of electric motors is achieved in the vast majority of commercial drives via nested loop PI controllers (Krause, 1986; Leonhardt, 1996; Parker Automation, 1998): the inner one wrapped around current errors and an external one that defines the desired values for these currents to generate a desired torque—for speed or position control. The rationale to justify this control configuration relies on the, often reasonable, assumption of time-scale separation between the electrical and the mechanical dynamics. In spite of its enormous success, to the best of our knowledge, a rigorous theoretical analysis of the stability of this scheme has not been reported. The main contribution of this paper is to (partially) fill-up this gap for the widely popular permanent magnet synchronous motors (PMSM), proving that the inner loop PI controller ensures global asymptotic stability (GAS) of the closed-loop, provided some viscous friction (possibly arbitrarily small) is present in the rotor dynamics, that the load torque is known and the proportional gain of the PI is suitably chosen, *i.e.*, sufficiently high. The assumption of known load torque is later relaxed proposing an adaptive scheme that, in the spirit of the aforementioned outer-loop PI, generates, via the addition of a simple integrator, an estimate for it—preserving GAS of the new scheme.

Several globally stable position and velocity controllers for PMSMs have been reported in the control literature—even in the sensorless context, *e.g.*, Bodson, Chiasson, Novotnak, and Ftekowski (1993), Lee et al. (2010), Tomei and Verrelli (2008, 2011) and references therein. However, these controllers have received an, at best, lukewarm reception within the electric drives community, which overwhelmingly prefers the aforementioned nested-loop PI configuration. Several versions of PI schemes based on fuzzy control, sliding modes or neural network control have been intensively studied in applications journals, see Jung, Leu, Do, Kim, and Choi (2015) for a recent review of this literature. To the best of our knowledge, a rigorous stability analysis of all these schemes is conspicuous by its absence.

The importance of disposing of a complete theoretical analysis in engineering practice can hardly be overestimated. Indeed, it gives the user additional confidence in the design and provides useful guidelines in the difficult task of commissioning the controller. The interest of our contribution is further enhanced by the fact that the analysis relies on basic Lyapunov theory, using the

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natural (quadratic in the increments) Lyapunov function. Various attempts to establish such a result for PMSMs have been reported in the literature either relying on linear approximations of the motor dynamics or including additional terms that cancel some nonlinear terms, see Hernandez-Guzman and Carrillo-Serrano (2011), Hernandez-Guzman and Silva (2011) and references therein—a standing assumption being, similarly to us, the existence of viscous friction.

The remainder of this paper is organized as follows. The models of the PMSM are given in Section 2. The problem formulation is introduced in Section 3. The passivity of the PMSMs incremental model and the PI controller are established in Section 4. The main stability results are provided in Section 5. Some concluding remarks and discussion of future research are given in Section 6.

Notation. For $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $A > 0$ we denote $|x|^2 = x^\top x$, $\|x\|_A^2 := x^\top A x$. For the distinguished vector $x^\star \in \mathbb{R}^n$ and a mapping $\mathcal{C} : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, we define the constant matrix $\mathcal{C}^\star := \mathcal{C}(x^\star)$.

Caveat Emptor. Due to page limitation constraints this is an abridged version of the full paper, which may be found in Ortega, Monshizadeh, Monshizadeh, Bazylev, and Pyrkin (2018).

2. Motor models

In this section we present the motor model, define the desired equilibrium and give its incremental model.

2.1. Standard dq model

The dynamics of the surface-mounted PMSM in the *dq* frame is described by Krause (1986) and Petrović, Ortega, and Stanković (2001):

$$\begin{aligned} L_d\frac{di_d}{dt} &= -R_s i_d + \omega L_q i_q + v_d \\ L_q\frac{di_q}{dt} &= -R_s i_q - \omega L_d i_d - \omega \Phi + v_q \\ J\frac{d\omega}{dt} &= -R_m\omega + n_p\left[(L_d-L_q)i_d i_q + \Phi i_q\right] - \tau_L \end{aligned} \tag{1}$$

where i_d, i_q are currents, v_d, v_q are voltage inputs, ω is the electrical angular velocity.[1] $\frac{2n_p}{3}$ is the number of pole pairs, $L_d > 0$, $L_q > 0$ are the stator inductances, $\Phi > 0$ is the back emf constant, $R_s > 0$ is the stator resistance, $R_m > 0$ is the viscous friction coefficient, $J > 0$ is the moment of inertia and τ_L is a constant load torque.

Defining the state and control vectors as

$$x := \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix},\ u := \begin{bmatrix} v_d \\ v_q \end{bmatrix}$$

the system (1) can be written in compact form as

$$\mathcal{D}\dot{x} + [\mathcal{C}(x) + \mathcal{R}]x = Gu + d,$$

where

$$\mathcal{D} := \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & \dfrac{J}{n_p} \end{bmatrix} > 0,\ \mathcal{R} := \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & \dfrac{R_m}{n_p} \end{bmatrix} > 0$$

$$\mathcal{C}(x) := \begin{bmatrix} 0 & 0 & -L_q x_2 \\ 0 & 0 & L_d x_1 + \Phi \\ L_q x_2 & -(L_d x_1 + \Phi) & 0 \end{bmatrix} = -\mathcal{C}^\top(x)$$

[1] Related with the rotor speed ω_m via $\omega = \frac{2n_p}{3}\omega_m$.

$$G := \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},\ d := \begin{bmatrix} 0 \\ 0 \\ -\dfrac{\tau_L}{n_p} \end{bmatrix},$$

Besides simplifying the notation, the interest of the representation above is that it reveals the power balance equation of the system. Indeed, the total energy of the motor is

$$H(x) = \frac{1}{2}x^\top \mathcal{D} x,$$

whose derivative yields

$$\underbrace{\dot{H}}_{\text{stored power}} = -\underbrace{x^\top \mathcal{R} x}_{\text{dissipated}} + \underbrace{y^\top u}_{\text{supplied}} - \underbrace{x_3\frac{\tau_L}{n_p}}_{\text{extracted}}, \tag{2}$$

where we used the skew-symmetry of $\mathcal{C}(x)$ and defined the currents as outputs, that is,

$$y := G^\top x = \begin{bmatrix} i_d \\ i_q \end{bmatrix}.$$

The current-feedback PI design is analysed in this paper viewing it as a passivity-based controller (PBC)—a term that was coined in Ortega and Spong (1989)—where the main idea is to preserve a power balance equation like the one above but now with a new stored energy and a new dissipation term. This objective is accomplished in two steps, the shaping of the systems energy to give it a desired form, *i.e.*, to have a minimum at the desired equilibrium, and the injection of damping. The shaped energy function qualifies, then, as a Lyapunov function that ensures stability of the equilibrium, which can be rendered asymptotically stable via the damping injection.

Remark 1. See Ortega, Loria, Nicklasson, and Sira-Ramirez (1998) and van der Schaft (2017) for additional discussion on the general theory of PBC and its practical applications and Aranovskiy, Ortega, and Cisneros (2016) and Zhang, Borja, Ortega, Liu, and Su (2018) for some recent developments on PID-PBC.

2.2. Incremental model

The industry standard desired equilibrium is the maximum torque per Ampere value defined as

$$x^\star := \mathrm{col}\left(0,\ \frac{1}{n_p\Phi}(\tau_L + R_m\omega^\star),\ \omega^\star\right), \tag{3}$$

where $\omega^\star$ is the desired electrical speed. With respect to this equilibrium we define the incremental model

$$\begin{aligned} \mathcal{D}\dot{\tilde{x}} + \mathcal{C}(x)\tilde{x} + [\mathcal{C}(x) - \mathcal{C}^\star]x^\star + \mathcal{R}\tilde{x} &= G\tilde{u} \\ \tilde{y} &= G^\top \tilde{x}, \end{aligned} \tag{4}$$

where $\tilde{(\cdot)} := (\cdot) - (\cdot)^\star$, $\mathcal{C}^\star := \mathcal{C}(x^\star)$, and we used the fact that

$$\begin{aligned} (\mathcal{C}^\star + \mathcal{R})x^\star &= Gu^\star + d \\ y^\star &= G^\top x^\star, \end{aligned}$$

with

$$u^\star = \begin{bmatrix} -\dfrac{1}{n_p\Phi}L_q\omega^\star(\tau_L + R_m\omega^\star) \\ \Phi\omega^\star + \dfrac{1}{n_p\Phi}R_s(\tau_L + R_m\omega^\star) \end{bmatrix}.$$

Note that

$$\tilde{y} = \begin{bmatrix} x_1 \\ x_2 - \dfrac{1}{n_p\Phi}(\tau_L + R_m\omega^\star) \end{bmatrix}. \tag{5}$$