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Brief paper

Tracking control of uncertain nonlinear systems with deferred asymmetric time-varying full state constraints☆

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ABSTRACT

In this paper, we investigate the tracking control problem of uncertain strict-feedback systems under deferred and asymmetric yet time-varying (DATV) constraints. We show that such type of constraints, occurring some time after (rather than from the beginning of) system operation, are frequently encountered in practice that have not been adequately addressed in existing works. By utilizing an error-shifting transformation, together with a new asymmetric Barrier Lyapunov Function with variational barrier bounds, we develop a tracking control method capable of dealing with DATV full state constraints under completely unknown initial tracking condition, leading to a control solution to the underlying problem. We also show that, with the proposed method, full state constraints being violated initially (rendering the previous methods inapplicable) can be made satisfied within a pre-specified finite time. The benefits and effectiveness of the proposed control are theoretically authenticated and numerically validated.

1. Introduction

Most practical systems operate under certain constraints due to physical limits or performance requirements, making the underlying control problem interesting yet challenging, which has enticed sustained research interest from control community during the past decades, resulting in a large number of reports on the subject, including those for output constraints (Niu & Zhao, 2013; Qiu, Liang, Dai, Cao, & Chen, 2015; Ren, Ge, Tee, & Lee, 2010; Tee, Ge, & Tay, 2009; Tee, Ren, & Ge, 2011; Zhao, Song, & Shen, 2018) and those for state constraints (Chen, Li, & Chen, 2017; He, Chen, & Yin, 2016; Jin, 2016; Liu, Li, & Tong, 2014; Liu, Li, Tong, & Chen, 2016; Liu, Lu, Li, & Tong, 2017; Liu & Tong, 2016, 2017; Song, Shen, He, & Huang, 2017; Wang, Wu, & Yu, 2017; Zhao, Song, Ma, & He, 2017).

However, with no exception, the established results are based on the implicit assumption that not only certain information on the initial tracking condition is available, but also the constraints are imposed from the beginning of system operation and are satisfied initially. In this work, we consider the more general and more interesting (although more challenging) situation that either the initial tracking condition is completely unknown and the output/state constraints are possibly not satisfied initially (we then need to regulate the outputs/states into the constrained boundary within prescribed finite time) or the output/state constraints, for some reason, are purposely imposed some time after the system being in operation. One immediate example involving such deferred constraints is that a mobile robot starts from a freely moving area for some time then enters a narrower (constrained) area for the purpose of, for instance, collision avoidance or target hiding. Another example is that a robotic arm, after some routine operation, is required to reach out to grasp an object along different paths or to fetch for a piece of tool in a container. One can find many other practical applications in which the deferred constraints are imposed (i.e., a group of aerial vehicles entering a tunnel, battleships passing through narrow straits etc.).

From control point of view, the above applications involve path tracking control with uncertain initial tracking conditions and deferred yet time-varying constraints, which in a more general case that has received little attention. The key difference and challenge, as compared to most existing works on constrained control, stem from the deferred constraints and the unknown initial tracking condition, rendering existing regular Barrier Lyapunov Function (BLF) based results invalid because the corresponding BLF is undefined initially or even for some period of time. In this work, we address the tracking control problem subject to this type of constraints, yet under unknown initial condition. More specifically, we explore a robust adaptive tracking control solution, under a variety of uncertain tracking conditions, for a class of strict-feedback nonlinear systems in the presence of deferred and asymmetric yet

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time-varying (DATV) constraints. Different from most existing results on constant constraints (Chen et al., 2017; He et al., 2016; Liu et al., 2014, 2016; Liu & Tong, 2016, 2017; Niu & Zhao, 2013; Ren et al., 2010; Tee et al., 2009; Wang et al., 2017; Zhao et al., 2017, 2018) or time-varying constraints but with known control gains (Liu et al., 2017; Qiu et al., 2015; Tee et al., 2011), here we consider more general DATV full state constraints for uncertain strict-feedback systems with unknown state-dependent control gains and external disturbances. To address the issue of unknown initial condition (including the initial constraint-violation) and the issue of DATV constraints, we introduce an error-shifting transformation and construct a new Asymmetric Barrier Lyapunov Function (ABLF), with which we establish a robust adaptive tracking control scheme (independent of initial tracking condition) to drive the states back into the restrictive boundary within pre-specified finite time, and the complexity of the controller design is reduced as compared to most existing methods with the commonly used piecewise ABLF (Liu et al., 2017; Qiu et al., 2015; Tee et al., 2011).

2. Problem formulation and preliminaries

2.1. Problem statement

Consider a class of strict-feedback systems with n state variables described by

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + d_i(\bar{x}_i, t), \quad i = 1, \ldots, n-1 \\ \dot{x}_n = g_n(\bar{x}_n)u + f_n(\bar{x}_n) + d_n(\bar{x}_n, t) \\ y = x_1 \end{cases} \tag{1}$$

subject to deferred and asymmetric constraints in that the states are free from any constraint during some initial period of time and then are fully constrained with time-varying boundaries starting from the time instant $t = T_c$, i.e.,

- $x_i(t)$ is free from constraint for $t \in [t_0, t_0 + T_c)$
- $x_i(t) \in (-\underline{k}_{ci}(t), \bar{k}_{ci}(t))$ for $t \in [t_0 + T_c, \infty)$

for $i = 1, \ldots, n$, where $\underline{k}_{ci}(t)$ and $\bar{k}_{ci}(t)$ are the pre-given time-varying boundaries on the system state variables, the specific forms of the boundary functions depend on actual requirements (i.e., security limits or performance considerations). $T_c > 0$ is the time instant at which all the state variables must obey the time-varying boundary conditions imposed, due to, for instance, safety and energy-saving consideration or physical limitation, etc. t_0 is the initial time of the system operation (without loss of generality, $t_0 = 0$ is used throughout this paper), $\bar{x}_i = [x_1, \ldots, x_i]^T \in R^i$, $i = 1, \ldots, n$ are measurable state vectors, $u \in R$ and $y \in R$ are the control input and system output, respectively, $g_i(\bar{x}_i)$ denotes the unknown state-dependent control coefficient, $f_i(\bar{x}_i)$ is an unknown but continuous nonlinear system function involving both parametric and nonparametric uncertainties, and $d_i(\bar{x}_i, t)$ represents the time-varying but bounded external disturbance.

The control objective is to design a robust adaptive control for system (1) under the aforementioned deferred asymmetric time-varying constraints such that the tracking error $z_1 = y - y_d$ converges to a small region contain origin for any bounded initial condition, and the deferred and time-varying full state constraints are strictly obeyed right after the finite time T_c.

Remark 1. In practice, a large class of plants can be described as or transformed into (1), such as mobile robot system, high speed trains system, and flexible crane system (Krstic, Kanelakopoulos, & Kokotovic, 1995; Liu et al., 2017). In contrast to those considered in most existing works, the problem here involves no constraints initially but time-varying and asymmetric full state constraints right after some time instant ($T_c > 0$). The situation that the constraints are possibly violated initially and need to be maintained within prescribed time can also be formulated as such deferred constraint control problem—the one that has been underexplored, although interesting and important.

To proceed, the following standard assumptions are imposed.

Assumption 1. It holds that $-\underline{y}_d(t) \le y_d(t) \le \bar{y}_d(t)$, where $\underline{y}_d(t)$ and $\bar{y}_d(t)$ are continuous positive functions, with $\underline{k}_{c1}(t) > \underline{y}_d(t)$ and $\bar{k}_{c1}(t) > \bar{y}_d(t)$. The signals $y_d(t), \underline{y}_d(t), \bar{y}_d(t)$, and their derivatives up to nth-order are known continuous and bounded. For $i = 1, \ldots, n$, $\underline{k}_{ci}(t)$ and $\bar{k}_{ci}(t)$ are C^{n-i+1}, and their derivatives up to $(n-i+1)$th-order are known and bounded.

Remark 2. Assumption 1 has been used in Jin (2016) and Liu et al. (2017) for nonlinear systems with time-varying full state constraints. Notice that $\underline{k}_{c1}(t) > \underline{y}_d(t)$ and $\bar{k}_{c1}(t) > \bar{y}_d(t)$ are always true in practice because the boundary requirement on the system output cannot be smaller than the bound on the desired output trajectory. The trajectories $y_d(t)$, $\underline{y}_d(t)$, and $\bar{y}_d(t)$, being differentiable up to nth-order, together with $\underline{k}_{ci}(t)$ and $\bar{k}_{ci}(t)$ for $i = 1, \ldots, n$, being C^{n-i+1}, is a commonly adopted assumption in tracking control literature using backstepping analysis (Jin, 2016).

Assumption 2. The control coefficients $g_i(\bar{x}_i), i = 1, \ldots, n$ are unknown and time-varying but bounded away from zero, i.e., there exist some unknown constants $\underline{g}_i$ and $\bar{g}_i$ such that $0 < \underline{g}_i \le |g_i(\bar{x}_i)| \le \bar{g}_i < \infty$, and without loss of generality, we further assume that all of the signs of $g_i(\bar{x}_i)$ are positive.

Assumption 3. Certain crude structural information on $f_i(\bar{x}_i)$ and $d_i(\bar{x}_i, t)$ is available to allow an unknown constant $c_i \ge 0$ and a known smooth function $\phi_i(\bar{x}_i) \ge 0$ to be extracted, such that $|f_i(\bar{x}_i) + d_i(\bar{x}_i, t)| \le c_i\phi_i(\bar{x}_i)$ for $t \ge 0$, where $\phi_i(\bar{x}_i)$ is either unconditionally bounded for any $\bar{x}_i$ in the domain of interest or bounded only if $\bar{x}_i$ is bounded.

Assumption 4. The strict-feedback nonlinear system (1) subject to parametric/nonparametric uncertainties and external disturbances as well as deferred state constraints is fully controllable such that feasible control schemes can be developed for such system to achieve the given control objective.

Remark 3. Assumption 2 is necessary for system (1) to be controllable and is commonly used in most existing works (Zhang, Xia, & Yi, 2017; Zhao et al., 2018). Assumption 3 is also widely used in robust adaptive control (Feng, Yu, & Man, 2002; Galicki, 2015; Song, Huang, & Wen, 2016; Wang, Song, & Lewis, 2015), where $\phi(\bar{x}_i)$ is referred to as the core function (Song et al., 2016; Wang, Song et al., 2015). See the example as shown in Song et al. (2016) for more details. In fact, $\phi(\bar{x}_i)$ can be easily derived (extracted) for practical systems with only crude model information by performing upper bound on $f_i(\bar{x}_i) + d_i(\bar{x}_i, t)$, see the simulation section for details. Assumption 4 is necessary to allow the system to admit feasible control solutions to the underlying tracking problem.

2.2. Shifting function

To deal with the unknown initial tracking conditions, we introduce the following shifting function,

$$\varphi(t) = \begin{cases} 1 - \left(\dfrac{T-t}{T}\right)^{n+2}, & 0 \le t < T \\ 1, & t \ge T \end{cases} \tag{2}$$

where $T > 0$ is a pre-specified finite settling time, and n is the system order or the number of the system state variables.

Remark 4. Two useful features are observed from the shifting function $\varphi(t)$ as defined in (2): first, $\varphi(0) = 0$; and second, $\varphi(t) = 1$ for $t \ge T$. If being used properly, the first feature could help