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Brief paper

On the construction of safe controllable regions for affine systems with applications to robotics☆

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ABSTRACT

This paper studies the problem of constructing in-block controllable (IBC) regions for affine systems. That is, we are concerned with constructing regions in the state space of affine systems such that all the states in the interior of the region are mutually accessible within the region's interior by applying uniformly bounded inputs. We first show that existing results for checking in-block controllability on given polytopic regions cannot be easily extended to address the question of constructing IBC regions. We then explore the geometry of the problem to provide a computationally efficient algorithm for constructing IBC regions. We also prove the soundness of the algorithm. We then use the proposed algorithm to construct safe speed profiles for robotic systems. As a case study, we present several experimental results on unmanned aerial vehicles (UAVs) to verify the effectiveness of the proposed algorithm; these results include using the proposed algorithm for real-time collision avoidance for UAVs.

1. Introduction

With the increasing desire for building the next generation of engineering systems that can safely interact with their environment and possibly non-professional humans (e.g., self-driving cars or assistive robots), there is an urgent need for developing controller design methods that respect all given safety constraints of the systems even in the transient period. Hence, we set our goal to provide the mathematical foundations for controller design under safety constraints. Although safety constraints can be accounted for using optimal/predictive control strategies (Aswani, Gonzalez, Sastry, & Tomlin, 2013; Qin & Badgwell, 2003), there are many fundamental questions in the area of controller design under safety constraints that still require further studies. For instance, consider a wheeled robot moving on a bounded table, with additional limits on the robot's speed. Using Kalman's controllability notion, we cannot even answer the simple question whether the robot can reach, starting from any initial position and speed, any final position and speed while respecting the safety constraints and using uniformly bounded input force? This shows the urgent need for finding checkable conditions for controllability under safety constraints.

Hence, we recently introduced the study of in-block controllability (IBC), which formalizes controllability under given safety state constraints (Helwa & Caines, 2014a, 2017). The notion of IBC can, however, be motivated from several different perspectives. In Helwa and Caines (2014c, 2015a), we showed that if one constructs a special partition (cover) of the state space of piecewise affine (PWA) hybrid systems (nonlinear systems) such that each region of the partition (cover) satisfies the IBC property, then one can systematically study controllability and build hierarchical structures for the PWA hybrid systems (nonlinear systems). We note that similar to nonlinear systems, controllability of PWA hybrid systems is a challenging open problem to date (Thuan & Camlibel, 2014). Also, building hierarchical structures of dynamical systems allows us to design controllers that achieve temporal logic statements at the higher levels of the hierarchy, and then to systematically realize these high-level control decisions at the lower levels. Moreover, the IBC notion is useful in the context of optimal control problems. In particular, the IBC conditions ensure that all the optimal accessibility problems within given safety constraints are feasible. Furthermore, in this paper we use the IBC results to build safe speed profiles for robotic systems. We then utilize these profiles to achieve obstacle avoidance and to determine the feasibility of given reference trajectories for the robots.

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The notion of IBC was utilized to build hierarchical structures of finite state machines, nonlinear systems on closed sets, and automata in Caines and Wei (1995, 1998), and Hubbard and Caines (2002), respectively. However, these papers do not study conditions for when the IBC property holds. In Helwa and Caines (2014a, 2017), three necessary and sufficient conditions were provided for IBC of affine systems on given polytopes. The conditions require solving linear programming (LP) problems at the vertices of the given polytope. In Helwa (2015), the IBC conditions were extended to controlled switched linear systems, while in Helwa and Caines (2014b), the notion of IBC was relaxed to the case where one can distinguish between soft and hard constraints. In Brammer (1972) and Sontag (1984), controllability of linear systems under input constraints was studied, while in Heemels and Camlibel (2007), controllability of continuous-time linear systems under state and/or input constraints was studied under the assumption that the system transfer matrix is right invertible.

In many practical scenarios, however, it may happen that the given affine system is not IBC with respect to (w.r.t.) the given polytope, representing the intersection of the given safety constraints. Hence, it would be important from a practical perspective to find the largest IBC region inside the given region, representing the largest safe region within which we can fully control our system. Also, constructing IBC regions is an essential problem for building the partitions/covers in Helwa and Caines (2014c, 2015a), so that one can make use of the hierarchical control results of these papers.

In this paper, we first show the difficulties of directly using the available results for checking IBC of affine systems on given polytopes to construct IBC regions. Second, we provide a computationally efficient algorithm for constructing IBC polytopes, and prove its soundness. Third, we show how our proposed algorithm can be useful for constructing safe speed profiles for robotic systems. That is, we construct for each position of the robot a corresponding safe speed range. The proposed safe speed profiles are useful for robot speed scheduling algorithms (Kant & Zucker, 1986; Ostafew, Schoellig, Barfoot, & Collier, 2014). If the speed scheduling algorithms limit the selected speeds to our proposed safe speed profiles, then safety of the robot can be always achieved on the given constrained position space by applying a feasible input within the actuation limits. We also show how the proposed safe speed profiles can be used to achieve static/dynamic obstacle avoidance. Moreover, our proposed algorithm guarantees full controllability of the robots on the constructed position–speed regions. Hence, in planning reference trajectories, it would be important to select reference points inside the proposed safe position–speed regions to ensure that they are reachable within the given state constraints and under the actuation limits. Finally, we verify our proposed results through several experimental results on unmanned aerial vehicles (UAVs). Compared to the brief version (Helwa & Schoellig, 2016), we hereby include complete proofs, additional discussions and remarks, and experimental results on UAVs.

Notation: Let $K \subset \mathbb{R}^n$ be a set. The closure of K is denoted by $\overline{K}$, the interior by K°, and the boundary by ∂K. For vectors $x,\ y \in \mathbb{R}^n$, $x \cdot y$ denotes the inner product of the two vectors. The notation $\|x\|$ denotes the Euclidean norm of x. The notation co $\{v_1, v_2, \ldots\}$ denotes the convex hull of a set of points $v_i \in \mathbb{R}^n$.

2. Related work

Compared to the well-known controlled invariance problem (Blanchini, 1999; Dorea & Hennet, 1999), which requires that all the state trajectories initiated in a set to remain in the set for all future time, IBC has the additional requirement of achieving mutual accessibility. Also, unlike the invariant sets, we guarantee that any state in the IBC set is reachable from any other state in the IBC set within the set itself, and consequently, any state in the IBC set can be selected as a point in a feasible reference trajectory for the system. In the literature, several algorithms have been provided for constructing controlled invariant sets. These algorithms can be classified into two main categories (Blanchini, 1999): *(i)* iterative algorithms for finding the largest invariant polytopic sets in given polytopes; these algorithms typically end up with polytopes with high complexity (Athanasopoulos, Bitsoris, & Lazar, 2014; Blanchini, 1999), and *(ii)* eigenstructure analysis algorithms leading to invariant polytopes with low complexity (Blanchini, 1999). Nevertheless, we emphasize that these algorithms cannot be used for building IBC regions, which are different from the invariant regions. Our proposed algorithm is not iterative, and it is based on exploring the geometric structure of the affine system, which has some similarities to the eigenstructure analysis algorithms for constructing invariant sets. Consequently, our algorithm is computationally efficient, and it ends up with polytopic regions with low complexity, which facilitates the construction of feedback laws on the constructed polytopes. For our geometric study of IBC, we utilize some geometric tools used for the study of the reach control problem (RCP) on polytopes; see Broucke (2010), Habets and van Schuppen (2004) and Helwa and Broucke (2013, 2015). Unlike RCP, in IBC, we do not try to force the trajectories of the system to leave the polytope through a prescribed exit facet.

Compared to the feasibility study of Schoellig, Hehn, Lupashin, and D'Andrea (2011), we hereby take the safety position/speed constraints into consideration in determining the feasibility of given references, and not only the robot actuation limits. In Section 6, we verify that the proposed algorithm is computationally efficient by utilizing it to build safe, controllable regions for UAVs online. Then, a control law is provided on the safe region to keep the system inside the region, and hence ensure dynamic obstacle avoidance. Collision avoidance strategies may be classified into: *(i)* motion planning strategies, and *(ii)* reactive control strategies (Rodriguez-Seda et al., 2014). Motion planning strategies calculate a collision-free reference trajectory at initial sampling time based on the estimated position of the obstacles. Fast replanning of collision-free trajectories is needed for dynamic environments (Grzonka, Grisetti, & Burgard, 2012). On the other hand, reactive control strategies continuously calculate updated control inputs online based on obstacles detected. Thus, these strategies are more suitable for fast-moving obstacles (Frew & Sengupta, 2004; Palafox & Spong, 2009; Rodriguez-Seda, Stipanovic, & Spong, 2011; Rodriguez-Seda et al., 2014). Our obstacle avoidance strategy for UAVs is a reactive one, and it has similarities to the strategy in Rodriguez-Seda et al. (2011) for fully-actuated robots and in Rodriguez-Seda et al. (2014) for nonholonomic, two-wheeled, ground vehicles. Unlike most of the obstacle avoidance approaches in the literature, see for instance (Frew & Sengupta, 2004; Palafox & Spong, 2009), our strategy takes the robot dynamics and actuation limits into account, and does not put constraints on the shape/velocity of the moving obstacle.

3. Background

We present some geometric background relevant for the remainder of the paper, see Rockafellar (1970). A set $K \subset \mathbb{R}^n$ is *affine* if $\lambda x + (1-\lambda)y \in K$ for all $x,\ y \in K$ and all $\lambda \in \mathbb{R}$. If the affine set passes through the origin, then it forms a subspace of $\mathbb{R}^n$. For subspaces $\mathcal{A},\ \mathcal{B}, \mathcal{A}+\mathcal{B} := \{a+b\ :\ a \in \mathcal{A},\ b \in \mathcal{B}\}$. The set $\mathcal{A}+\mathcal{B}$ is also a subspace. The *affine hull* of a set K, denoted by aff (K), is the smallest affine set containing K. We mean by a dimension of a set K its affine dimension, which is the dimension of aff (K). A *hyperplane* is an $(n-1)$-dimensional affine set in $\mathbb{R}^n$, dividing $\mathbb{R}^n$ into two open half-spaces. A finite set of vectors $\{x_1, \ldots, x_k\}$ is called *affinely independent* if the unique solution to $\sum_{i=1}^{k}\alpha_i x_i = 0$ and $\sum_{i=1}^{k}\alpha_i = 0$ is $\alpha_i = 0$ for all $i = 1, \ldots, k$. Affinely independent vectors do