ARTICLE IN PRESS

Journal of Sound and Vibration xxx (2018) 1–17

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Some benefits of using exact solutions of forced nonlinear oscillators: Theoretical and experimental investigations

Ivana Kovacic [a, *], Gianluca Gatti [b]

[a] University of Novi Sad, Faculty of Technical Sciences, Centre of Excellence for Vibro-Acoustic Systems and Signal Processing CEVAS, 21000 Novi Sad, Serbia

[b] University of Calabria, Department of Mechanical, Energy and Management Engineering, Cosenza, Italy

A R T I C L E I N F O

Article history:
Received 15 March 2018
Received in revised form 4 June 2018
Accepted 26 June 2018
Available online xxx
Handling Editor: C-H Lamarque

Keywords:
Simple harmonic oscillator
Duffing-type oscillators
External excitation
Tuning

A B S T R A C T

This work first presents an analytical technique on how to exploit exact solutions for the response of certain oscillators to design specific external excitation and get a desired form of the exact steady-state response, both in the undamped and damped systems. Related benefits are then discussed, which include: (i) producing the response of free nonlinear oscillators by exciting a linear system in a particular way, (ii) determining the multi-term excitations of nonlinear oscillators to produce a single-term harmonic response, and (iii) tuning nonlinearities in a system excited by certain two-term excitation to produce a single-term harmonic response. Illustrations of these applications are presented in terms of physical and computer-aided experiments.

1. Introduction

Many perturbation and non-perturbation techniques have been developed during the previous decades to obtain analytical solutions for the responses of externally (mainly harmonically) excited nonlinear oscillators and detailed insights into their dynamics (see, for example [1], and [2]). All of them have been approximate, and, thus, associated either with certain limitations or required some validation, such as, for example, numerical confirmations. However, recent investigations have led to a different strategy, concerned with the determination of the exact solutions for the steady-state response of nonlinear oscillators and the design of tuned external excitations that will produce it [3–7]. The idea of applying specially designed external excitation to obtain the exact solution for the resulting steady-state response dates back to Hsu [8]. This idea has recently been extended to forced one-degree-of-freedom undamped nonlinear oscillators with cubic and quadratic nonlinearities [3], pure real-powered nonlinearities [4,5], multi-degree-of-freedom purely nonlinear chains [6], as well as to damped Duffing oscillators [9] and a variety of other damped nonlinear oscillators [7].

Some of the benefits of knowing exact solutions for free and forced response of nonlinear oscillators with certain multi-term external excitations include:

Case 1. Designing external excitations of linear oscillators so that their steady-state response corresponds to a free response of certain nonlinear oscillators. Let us imagine the following situation: a teacher/researcher brings students/colleagues in a lab, aiming to demonstrate different oscillatory responses. However, there is only a linear spring available, and the intention is

* Corresponding author.
E-mail address: ivanakov@uns.ac.rs (I. Kovacic).

https://doi.org/10.1016/j.jsv.2018.06.059
0022-460X/

to demonstrate the responses of Duffing oscillators as they represent archetypical nonlinear oscillators. This study shows how this could be possible without designing a special geometrical configuration of the spring, but by adding a specially designed external excitation to a simple linear oscillator. In addition, this case could be useful as it will allow the implementation of a feed-forward control strategy to excite a linear electro-mechanical actuator and get the desired response, as an alternative to the classical feed-back control strategy.

Case 2. Designing external excitations of certain nonlinear oscillators whose steady-state response has only one harmonic, as in a free simple harmonic oscillator. It is well-known that when a nonlinear oscillator is excited by a single-harmonic external excitation, its resulting response is, in general, multi-harmonic. Higher harmonics are often neglected, which can affect the related accuracy and reliability. The strategy presented in this paper will demonstrate theoretically and experimentally how to design the external excitation so that only one harmonic appears in the response without any approximation or omission.

Case 3. Tuning nonlinearities in a system excited by certain two-term excitation to produce a single-term harmonic. Note that as in Case 2, the multi-term excitation and nonlinearity will, in general, result in a multi-harmonic response. However, it will be shown in this paper how higher-order terms can be analytically removed. This case is actually the inverse one with respect to Case 2, as the tuning of nonlinear terms is done depending on the excitation, while in Case 2 tuning of the excitation is done depending on the existing nonlinearity.

The aim of this work is thus to illustrate, for the first time, the above-mentioned Cases 1 - 3 by using physical and computer-aided experiments, as these realizations are believed to be useful for engineering design, and to stimulate further ideas for practical utilizations.

2. Case 1: Externally excited linear oscillators responding as free nonlinear oscillators

2.1. Analytical approach

A particle of mass m attached to a linear spring of stiffness k behaves as a simple harmonic oscillator (SHO)

$$\ddot{x} + \omega^2 x = F, \tag{1}$$

where $\omega = \sqrt{k/m}$ is its natural frequency and x is the displacement of the mass. The term F that appears on the right-hand side of Eq. (1) represents the external excitation and it is assumed herein to be proportional to the displacement and its cube as follows

$$F = Bx + Dx^3, \tag{2}$$

where B and D are constants, so that Eq. (1) becomes

$$\ddot{x} + \left(\omega^2 - B\right)x - Dx^3 = 0. \tag{3}$$

Depending on the sign and values of the coefficients in front of the linear and cubic terms, this equation can correspond to several classical free undamped Duffing-Type Oscillators (DTOs), which can be represented in the form

$$\ddot{x} + c_1 x + c_3 x^3 = 0. \tag{4}$$

This work encompasses:

i) Hardening Duffing Oscillators (HDOs), when $c_1 > 0$ and $c_3 > 0$;
ii) Softening Duffing Oscillators (SDOs), when $c_1 > 0$ and $c_3 < 0$;
iii) Pure Cubic Oscillators (PCOs), when $c_1 = 0$ and $c_3 < 0$;
iv) Bistable Duffing Oscillators (BDOs), when $c_1 < 0$ and $c_3 > 0$.

What is of importance for this study is the fact that these DTOs have their solutions for motion in the known and exact form, as detailed in Ref. [10]. These solutions can be expressed in terms of the Jacobi elliptic function (labeled by ep here), as

$$x_{\mathrm{DTO}} = A\,\mathrm{ep}(\omega_{\mathrm{DTO}} t, k_{\mathrm{DTO}}), \tag{5}$$

which is a two-parameter function, having the first argument varying in time t and proportional to the frequency ω_{DTO}, while the second argument is the so-called elliptic modulus k_{DTO} [2,10] (note that in some publications and software packages, the elliptic parameter m_{DTO} is used, where $m_{\mathrm{DTO}} = k_{\mathrm{DTO}}^2$). The exact solutions for different DTOs are collected in Table 1. This table also contains sketches of the corresponding phase trajectories and the fixed points around which these oscillations take place