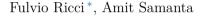
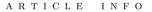




## Spherical analysis on homogeneous vector bundles $\stackrel{\Rightarrow}{\sim}$



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Article history: Received 27 September 2017 Received in revised form 9 September 2018 Accepted 9 September 2018 Available online 26 September 2018 Communicated by the Managing Editors

MSC: 43A90 43A85

Keywords: Spherical functions Spherical transforms Gelfand pairs Homogeneous bundles

## ABSTRACT

Given a Lie group G, a compact subgroup K and a representation  $\tau \in \widehat{K}$ , we assume that the algebra of  $\operatorname{End}(V_{\tau})$ -valued, bi- $\tau$ -equivariant, integrable functions on G is commutative. We present the basic facts of the related spherical analysis, putting particular emphasis on the rôle of the algebra of G-invariant differential operators on the homogeneous bundle  $E_{\tau}$  over G/K. In particular, we observe that, under the above assumptions, (G, K) is a Gelfand pair and show that the Gelfand spectrum for the triple  $(G, K, \tau)$  admits homeomorphic embeddings in some  $\mathbb{C}^n$ .

In the second part, we develop in greater detail the spherical analysis for  $G = K \ltimes H$  with H nilpotent. In particular, for  $H = \mathbb{R}^n$  and  $K \subset SO(n)$  and for H equal to the Heisenberg group  $H_n$  and  $K \subset U(n)$ , we characterize the representations  $\tau \in \hat{K}$  giving a commutative algebra.

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## 0. Introduction

Let (G, K) be a Gelfand pair with G a Lie group and K a compact subgroup of it. Recent work has put attention on the fact that the spherical analysis on the bi-K-invariant

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https://doi.org/10.1016/j.aim.2018.09.016



 $<sup>^{\</sup>star}$  This research has been supported by the Italian MIUR PRIN Grant Real and Complex Manifolds: Geometry, Topology and Harmonic Analysis, 2010–2011.

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algebra  $L^1(K \setminus G/K)$  gains new interesting aspects from the fact that its Gelfand spectrum  $\Sigma$ , i.e., the space of bounded spherical functions with the compact-open topology, can be naturally embedded into some Euclidean space as a closed set [8].

Such an embedding  $\rho$  is defined by choosing a generating k-tuple  $(D_1, \ldots, D_k)$  in the algebra of G-invariant differential operators on G/K and assigning to the spherical function  $\phi$  on G/K the vector  $\rho(\phi) = (\lambda_{D_1}(\phi), \ldots, \lambda_{D_k}(\phi)) \in \mathbb{C}^k$  whose entries  $\lambda_{D_j}(\phi)$ are the eigenvalues of  $\phi$  under the  $D_j$ 's.

This allows to introduce a notion of smoothness for functions defined on  $\Sigma$ , and to pose the problem, classical in Fourier analysis, of relating smoothness of the spherical transform of a given bi-K-invariant function on G with properties of the function itself. This question has been investigated in detail for *nilpotent Gelfand pairs*, in which  $G = K \ltimes H$  is a motion group on a nilpotent group H [9–11].

In this paper we extend the basic framework for such analysis to the spherical transform of type  $\tau$ , where  $\tau$  is an irreducible unitary representation of K for which the appropriate commutativity assumptions are satisfied.

The notion of spherical transform of type  $\tau$  goes back to [15]. In most of the existing literature the accent is on the case where (G, K) is a symmetric pair. For the general case we refer to [28, Ch. 6] and [25,3].

There are at least two equivalent ways to introduce the objects of our analysis on a triple  $(G, K, \tau)$ , where G is a Lie group, K a compact subgroup of it and  $\tau \in \hat{K}$  as above.

In the first (matrix-valued) picture one considers the homogeneous bundle  $E_{\tau} = G \times_{\tau} V_{\tau}$  with basis G/K and linear operators on sections of  $E_{\tau}$  commuting with the action of G. The Schwartz kernel theorem implies that, under mild continuity assumptions, any such operator can be represented by convolution with an  $\text{End}(V_{\tau})$ -valued<sup>1</sup> distributional kernel F on G satisfying the identity

$$F(k_1 x k_2) = \tau(k_2^{-1}) F(x) \tau(k_1^{-1}).$$
(0.1)

The commutativity condition imposed on  $\tau$  is that the algebra of  $End(V_{\tau})$ -valued integrable functions F on G satisfying the above identity is commutative with respect to the convolution defined in (1.4) below.

In the second (scalar-valued) picture one considers the algebra of integrable scalarvalued functions f on G which are K-central and satisfy the identity

$$f * \overline{\chi}_{\tau} = f,$$

and the requirement on  $\tau$  is that this algebra be commutative.

We say that  $(G, K, \tau)$  is a *commutative triple* if either of these conditions is satisfied.

<sup>&</sup>lt;sup>1</sup> For V, W finite dimensional complex vector spaces,  $\operatorname{Hom}(V, W)$  denotes the space of linear operators from V to W. If V = W, we write  $\operatorname{End}(V)$  instead of  $\operatorname{Hom}(V, V)$ .

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