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# An application of Pfaffians to multipeakons of the Novikov equation and the finite Toda lattice of BKP type

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ABSTRACT

The Novikov equation is an integrable analogue of the Camassa–Holm equation with a cubic (rather than quadratic) nonlinear term. Both these equations support a special family of weak solutions called multipeakon solutions. In this paper, an approach involving Pfaffians is applied to study multipeakons of the Novikov equation. First, we show that the Novikov peakon ODEs describe an isospectral flow on the manifold cut out by certain Pfaffian identities. Then, a link between the Novikov peakons and the finite Toda lattice of BKP type (B-Toda lattice) is established based on the use of Pfaffians. Finally, certain generalizations of the Novikov equation and the finite B-Toda lattice are proposed together with special solutions. To our knowledge, it is the first time that the peakon problem is interpreted in terms of Pfaffians.


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## 1. Introduction

It is essential to search for simple mathematical models for effectively describing nonlinear phenomena in nature, for example, the breakdown of regularity. Whitham, in his book [48], emphasized the need for a water wave model exhibiting a soliton interaction, the existence of peaked waves, and, at the same time, allowing for breaking waves. In 1993, Camassa and Holm [10] derived such a shallow water wave model

$$m_t + (um)_x + mu_x = 0, \qquad m = u - u_{xx}, \tag{1.1}$$

nowadays known as the Camassa–Holm (CH) equation. The CH equation admits, in particular, peakon solutions (simply called peakons) as its solitary wave solutions with peaks. In the case of $n$ peaks the solution takes the form

$$u = \sum_{j=1}^{n} m_j(t) e^{-|x-x_j(t)|}.$$

Clearly, the peakons are smooth solutions except at the peak positions $x = x_j(t)$ where the $x$ derivative of $u$ is discontinuous, forcing us to interpret peakons in a suitable weak sense. The mathematics of peakons has attracted a great deal of attention since they