Accepted Manuscript

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V.N. Temlyakov

PII: S0021-9045(18)30103-5
DOI: https://doi.org/10.1016/j.jat.2018.09.002
Reference: YJATH 5234

To appear in: *Journal of Approximation Theory*

Received date : 5 November 2017
Revised date : 22 June 2018
Accepted date : 4 September 2018

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V.N. Temlyakov*

Abstract

It is proved that the Fibonacci and the Frolov point sets, which are known to be very good for numerical integration, have optimal rate of decay of dispersion with respect to the cardinality of sets. This implies that the Fibonacci and the Frolov point sets provide universal discretization of the uniform norm for natural collections of subspaces of the multivariate trigonometric polynomials. It is shown how the optimal upper bounds for dispersion can be derived from the upper bounds for a new characteristic – the smooth fixed volume discrepancy. It is proved that the Fibonacci point sets provide the universal discretization of all integral norms.

1 Introduction

The concept of *dispersion* of a point set is an important geometric characteristic of a point set. It was established in a recent paper [21] that the property of a point set to have the minimal in the sense of order dispersion is equivalent, in a certain sense, to the property of the set to provide universal discretization in the L_∞ norm for natural collections of subspaces of the multivariate trigonometric polynomials. In this paper we study decay of dispersion of the Fibonacci and the Frolov point sets with respect to the cardinality of sets. We remind the definition of dispersion. Let $d \geq 2$ and $[0,1)^d$ be the d-dimensional unit cube. For $\mathbf{x}, \mathbf{y} \in [0,1)^d$ with $\mathbf{x} = (x_1, \dots, x_d)$ and

*University of South Carolina, Steklov Institute of Mathematics, and Lomonosov Moscow State University.