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The immersed discontinuous semi-discrete finite element method: A comparison of surface-immersed stiffened and body-immersed stiffened sandwich plate

Tianyu Li[a,*], Hua Kang[b]

[a] Aerospace and Ocean Engineering, Virginia Tech, United States
[b] Civil and Environmental Engineering, UCLA, United States

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ABSTRACT

Two types of stiffened sandwich plate structures are analyzed and compared. The first is surface-immersed stiffened sandwich plate, in which the stiffener's top surface is bonded to the bottom surface of the sandwich plate. The second is body-immersed stiffened sandwich plate, in which the stiffener is immersed into the 3-D space of core soft layer. The immersed discontinuous semi-discrete finite element method is developed and applied to study the stiffened plates. The results are compared with ANSYS to verify the accuracy of the method. The convergence study is also given: an exponential convergence is observed. The stiffening efficiency is compared between the two types of stiffened sandwich plates. It is observed that the body-immersed stiffened plate has better strength and stability. The motivation to develop this method is to use a much smaller number of degree-of-freedom to analyze those stiffened plates, for which traditional finite element methods may not be convenient because of the difficulty in the generation of high-quality compatible mesh.

1. Introduction

The stiffened sandwich plate is studied in this paper. Two different types of stiffened plates are analyzed: the surface-immersed stiffened and the body-immersed stiffened, as Fig. 1 shows. The motivations of the paper are: (a) to compare the stiffening efficiency of the two types of stiffened plates; (b) to develop an highly efficient immersed semi-discrete discontinuous finite element method for these stiffened structures.

For the stiffened plate structures, traditional finite element methods may not be suitable because the shape of the stiffeners could be arbitrary and a highly-qualified compatible mesh is not so easy to obtain. Moreover, the stiffener is always a slender curved beam. In this case, the thickness of stiffener is much smaller than the characteristic size of the plate. Thus, the total number of elements must be very large if an acceptable mesh quality is expected. Based on these considerations, the immersed discontinuous finite element is a good choice to analyze the stiffened plate structure. For the immersed-type finite element method, compatible mesh is not needed. The kinematics and displacement field of plates and stiffeners are independent. The displacement compatibility condition between stiffeners and plates is satisfied in the weak form.

In order to reduce the total number of degree-of-freedom, it is noted that the proposed method is only discrete in the plate's thickness direction. In the plate membrane dimensions, the Gelerkin Ritz idea is used and the displacement unknown is defined globally based on the Consistent Orthogonal Basis Function Space [1,2]. In this case, the total number of degree-of-freedom is much smaller than that in traditional finite element methods. For the stiffened, a novel kinematics is developed such that a global displacement can be defined. For this kinematics theory, the configuration of arbitrarily-shaped stiffener is smooth and exactly accurate. In the aspect of configuration, traditional finite element methods use many small C0 elements to represent a complicated structure, which has discretization error in the structure configuration. For this paper's method, there is no discretization error because the structure configuration is globally continuous and exactly defined.

By using this method, two types of stiffened sandwich plates are analyzed. The results are compared. The first type is surface-immersed stiffened plate and the second is body-immersed stiffened plate. For the surface-immersed stiffened plate, the stiffener's top surface is bonded to the bottom surface of the plate. For the second type, the stiffener is immersed into the 3-D space of the core layer of the sandwich plate. We would like to compare the response for these two types of stiffened

* Corresponding author.
E-mail address: lty1990@vt.edu (T. Li).

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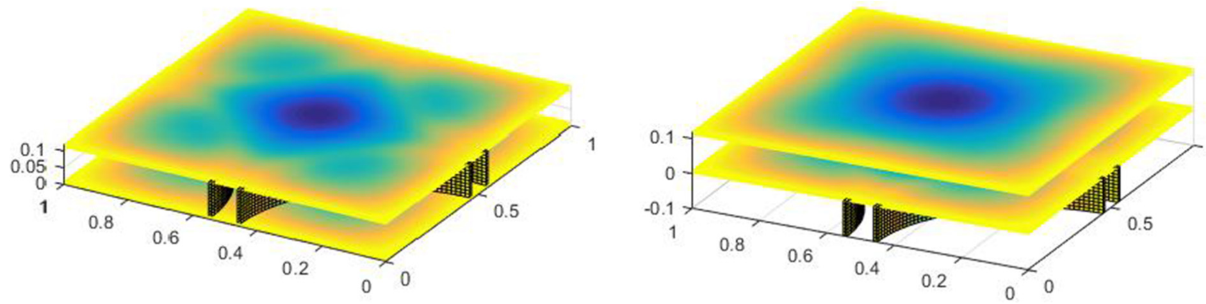

Fig. 1. The transverse deflection contours under uniform pressure for: (a) body-immersed stiffened sandwich plate and (b) the surface-immersed stiffened plate.

plates:

Which one has better strength? Which one has better stability?

These two properties (strength and stability) are very important to evaluate the structure.

The literature review of stiffened structures and related finite element applications are presented here. The proposed method is based on the laminated plate finite element developed in [1]. In [1,2], the basis functions are developed such that the basis functions are very identical to mode shapes. The nonlinear finite element methods are very important for solid mechanics problems [1–4]. In [5], the sandwich plate with multiple stiffeners are analyzed with an analytical solution. In [6], the free vibration and stability analysis is given for stiffened plate. The finite trip method is applied based on B-splines. In [7], finite element analysis is given for both non-stiffened plate and stiffened plate. The impact analysis for sandwich plate is given in [8]. For more papers about sandwich plates/shells and finite element, it is referred to [9,10].

In [11], the immersed finite element is developed and applied to solve the fluid–structure-interaction problem. The mesh of structures and fluids can be non-conforming and the coupling condition between structures and fluids is satisfied in weak form. In [12], the various biomechanics problems are solved by immersed finite element. In [13], the immersed finite element is studied in the aspect of PDE.

The literature review is given here. In [14], the analysis of collar plate and doubler plate reinforced SHS T-joints under axial compression is presented. In [15], the differential quadrature method is applied to analyze the composite plates with nanotube. In [16], the nonlinear vibration of nanotude-reinforced plate is presented. In [17], the Ritz method is applied to analyze the composite plates with stiffeners. In [18], the Piezoelectric composite plate with reinforced-nanotube is focused. Carbon nanotube reinforced plate is an important topic among composite plates and more literature in this field is referred to [19]. In [21], a higher order layer-wise theory is developed for the analysis of interlaminar shear stress. The proposed method in this paper could be used to analyze these stiffened plate problems.

The outline of the paper is given. In Section 2, the kinematics of arbitrary curve stiffener is presented. In Section 3, the weak form finite element equation for the immersed finite element method is derived. In Section 4, the basis functions are discussed. In Section 5, the nonlinear finite element implementation details are presented. In Section 6, the numerical tests for the surface-immersed stiffened sandwich plate and the body-immersed stiffened sandwich plate are given. In Section 7, the conclusion is given.

2. Kinematics of sandwich plate and arbitrary stiffeners

In this section, the kinematics of sandwich plate and the stiffener are presented. Throughout this paper, it is assumed that the shape of the stiffener can be arbitrary, as Fig. 2 shows.

2.1. Kinematics of arbitrary stiffener

For the kinematics of arbitrary stiffener, it is presented in Fig. 2. The

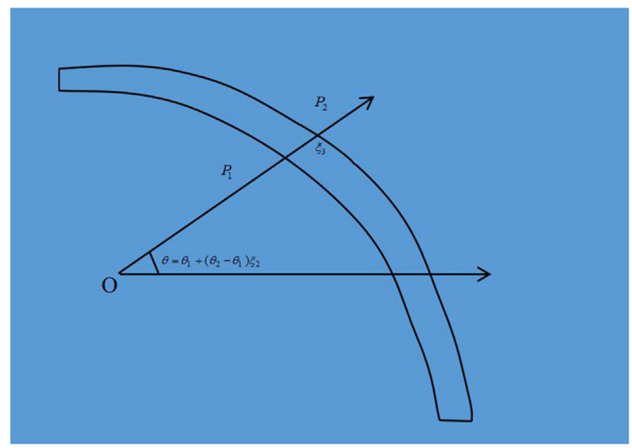

Fig. 2. The Kinematics of arbitrary shape of stiffener.

origin O is placed on the plate plane such that the whole stiffener could be seen from point O. Assume there is a radial line orienting from point O, the radial line is identified by the angle θ with the horizontal direction. The radial line will interact with the stiffener's inner surface and outer surface at points P_1, P_2, respectively. Thus, we can define the following two distance functions as:

$$R_1 = dist(OP_1) = R_1(\theta)$$
$$R_2 = dist(OP_2) = R_2(\theta) \quad \text{(2-1a,b)}$$

where $dist(O, P_1)$ is the distance between point O and point P_1, $dist(O, P_2)$ is the distance between point O and point P_2.

Assume that the angle belongs to the interval:

$$\theta \in [\theta_1, \theta_2] \quad \text{(2-2)}$$

where $[\theta_1, \theta_2]$ is the interval of θ.

Then, the natural coordinate ξ_2 is defined as:

$$\xi_2 = \frac{\theta - \theta_1}{\theta_2 - \theta_1} \in [0, 1] \quad \text{(2-3)}$$

Thus, the distance function could be re-written as:

$$R_1 = dist(OP_1) = R_1(\xi_2)$$
$$R_2 = dist(OP_2) = R_2(\xi_2) \quad \text{(2-4a,b)}$$

Assume that the stiffener has the height of h_s, thus, the configuration of the arbitrary curved stiffener could be represented as:

$$\vec{r_s} = \begin{bmatrix} (R_1(\xi_2) + \xi_3(R_2(\xi_2) - R_1(\xi_2)))\sin(\theta_1 + (\theta_2 - \theta_1)\xi_2) \\ (R_1(\xi_2) + \xi_3(R_2(\xi_2) - R_1(\xi_2)))\cos(\theta_1 + (\theta_2 - \theta_1)\xi_2) \\ h_s\xi_1 \end{bmatrix} \quad \text{(2-5)}$$

where $\xi_i \in [0, 1]$, $i = 1, 2, 3$ are the natural coordinates for the arbitrary curved stiffener structure. By using different functions of R_1 and R_2, various curved stiffener with arbitrary shape could be represented.