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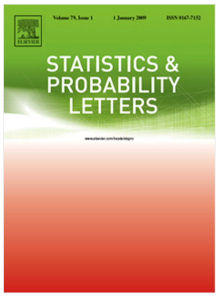

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Abstract

We obtain upper and lower bounds for the probability that random walk leaves the strip through the upper boundary.

Key words: random walk, two-sided boundary crossing problem, ruin probability, sequential probability ratio test.

Let $X_1, X_2, \ldots$ be independent identically distributed random variables with common nondegenerate distribution, and $S_0 = 0$, $S_n = X_1 + \cdots + X_n$.

Given arbitrary $a > 0$ and $b > 0$, we introduce the random variable

$$N = N_{a,b} = \inf\{n \geq 1 : S_n \notin (-a, b)\},$$

equal to the first exit time from the interval $(-a, b)$ for the random walk, and let

$$\alpha(a, b) = \mathbf{P}(S_N \leq -a), \quad \beta(a, b) = \mathbf{P}(S_N \geq b).$$

These both quantities are usually called the ruin probabilities in the game of two players; here we obviously have $\alpha(a, b) + \beta(a, b) = 1$.

In the paper, we obtain upper and lower bounds for the probability $\beta(a, b)$.

The first steps in the study of the ruin probability were made as early as XVII century (see the survey of Takács, 1969), while considering game situations, and were restricted to the case when $\mathbf{P}(X_1 = -1) + \mathbf{P}(X_1 = 1) = 1$. Later it turned out that many other significant problems lead to random walks with two boundaries; here we mention finding error probabilities and operating characteristics of the sequential probability ratio test (Wald, 1947, Siegmund, 1985), change point detecting by the procedure of cumulative sums (CUSUM procedure, Page, 1954), the study of various characteristics of queues, etc.

The exact calculation of $\beta(a, b)$ is available only in some particular situations. This is why the accent in the study shifted to finding approximating formulas and, in particular, to the study of the asymptotic behavior of the ruin probability in the cases when asymptotic analysis is possible (e.g., in a triangular array scheme (Nagaev, 1971) or if $a + b \to \infty$ (Lotov, 1988a, 1988b, 1991, 1999), Siegmund, 1985).

The derivation of upper and lower bounds for the ruin probability is a natural complement to the approximating results.

1 Random walks with negative drift

Suppose that there exists $\mathbf{E}X < 0$. Put

$$S = \sup_{n \geq 0} S_n, \quad \mathbf{P}(S \geq x) = Q(x).$$