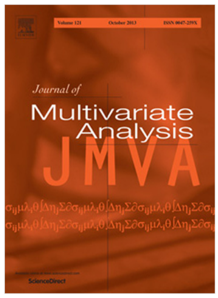
Accepted Manuscript



On parameter estimation of the hidden Ornstein–Uhlenbeck process

Yury A. Kutoyants

PII: S0047-259X(18)30045-9
DOI: https://doi.org/10.1016/j.jmva.2018.09.008
Reference: YJMVA 4418

To appear in: *Journal of Multivariate Analysis*

Received date : 23 January 2018

On parameter estimation of the hidden Ornstein–Uhlenbeck process

Yury A. Kutoyants

Le Mans University, Le Mans, France, and Tomsk State University, Tomsk, Russia

Abstract

This paper considers parameter estimation in the Ornstein–Uhlenbeck process observed in the presence of Gaussian white noise. We show the consistency and asymptotic normality of the maximum likelihood estimator in small-noise asymptotics. The data are assumed to arise from a non-homogeneous partially observed linear system. The construction and study of the estimator are based mainly on the asymptotics of the equations of Kalman–Bucy filtration.

Keywords: Hidden process, Parameter estimation, Partially observed linear system, Small noise asymptotics
2000 MSC: 62M02, 62G10, 62G20.

1. Introduction

Consider a non-homogeneous partially observed linear system described by the equations

$$\mathrm{d}X_t = f(\vartheta, t)\, Y_t \mathrm{d}t + \varepsilon\sigma(t)\, \mathrm{d}W_t, \quad X_0 = 0, \tag{1}$$

$$\mathrm{d}Y_t = -a(\vartheta, t)\, Y_t \mathrm{d}t + b(\vartheta, t)\, \mathrm{d}V_t, \quad Y_0 = y_0 \neq 0, \tag{2}$$

where f, σ, a and b are known, smooth functions, while $(W_t, 0 \leq t \leq T)$ and $(V_t, 0 \leq t \leq T)$ are two independent Wiener processes. We concentrate on the estimation of the one-dimensional parameter $\vartheta \in \Theta = (\alpha, \beta)$ from continuous time observations $X^T = (X_t, 0 \leq t \leq T)$, given that the process $(Y_t, 0 \leq t \leq T)$ is unobservable (hidden).

Model (1)–(2) is typically used in the Kalman–Bucy filtration [1, 16, 23], which provides a closed form system of equations describing the conditional expectation $m(\vartheta, t) = \mathrm{E}_\vartheta (Y_t | X_s, 0 \leq s \leq t)$. Statistical issues for discretely observed hidden Markov processes were studied by many authors; see, e.g., [2, 3, 6, 7] and references therein. However, the literature on continuous time models is limited. In a continuous time context, we refer the reader to [20] for linear and non-linear partially observed systems with small noise, to [6] for continuous-time hidden Markov models estimation, as well as to [5, 18] for the hidden telegraph process observed in the Gaussian white noise.

In the present paper we focus on the asymptotic behavior of the maximum likelihood estimator (MLE) $\hat{\vartheta}_\varepsilon$ and Bayesian estimator (BE) $\tilde{\vartheta}_\varepsilon$ in small-noise asymptotics, i.e., when $\varepsilon \to 0$. The statistical problems associated with such observation models have been widely studied, owing to the many engineering applications of Kalman–Bucy filtration. This problem could be considered as a parameter estimation problem for the hidden process Y_t. The behavior of diffusion processes with small noise was studied in [8] and related statistical issues were treated in [20].

Let us now give the definitions of the MLE and BE. First we present conditions C_1, C_2 which provide the equivalence of the measures $\{\mathbf{P}_\vartheta, \vartheta \in \Theta\}$ induced by the observations (1) in the measurable space of continuous functions on $[0, T]$. The likelihood ratio function [23] is given, for all $\vartheta \in \Theta$, by

$$V(\vartheta, X^T) = \exp\left\{ \int_0^T \frac{f(\vartheta, t)\, m(\vartheta, t)}{\varepsilon^2 \sigma(t)^2}\, \mathrm{d}X_t - \int_0^T \frac{f(\vartheta, t)^2\, m(\vartheta, t)^2}{2\varepsilon^2 \sigma(t)^2}\, \mathrm{d}t \right\}.$$

Then the MLE $\hat{\vartheta}_\varepsilon$ is defined by the relation

$$V(\hat{\vartheta}_\varepsilon, X^T) = \sup_{\vartheta \in \Theta} V(\vartheta, X^T).$$