# Numerical evaluation of coupled galloping of slender towers in boundary-layer winds based on a nonlinear analytical model

Liang Wu, Delong Zuo *

*Department of Civil, Environmental and Construction Engineering, Texas Tech University, Lubbock, TX, USA*

A B S T R A C T

Slender towers with certain shapes of cross-section can exhibit galloping that involves coupling between the oscillation components about the principal structural axes. This study presents a nonlinear analytical model of this type of galloping for towers subjected to winds in the atmospheric boundary layer. The formulation of the model adopts the assumption of quasi-steady wind loading. However, this formulation differs from those for the classical across-wind galloping in that it does not require the wind to be along the direction of a principal structural axis. In addition, it considers the oscillation components about both structural axes, instead of only the across-wind oscillation, to enable the evaluation of the coupling between the components. Further, because the formulation does not linearize the wind-induced force acting on the structure based on the assumption of small vibration amplitude, the resultant model can be used as a basis to numerically assess the amplitudes and frequencies of large-amplitude coupled galloping oscillations. In an illustrative application, the model is used to evaluate the wind-induced vibration of a full-scale slender tower. The wind fields are simulated using the spectral representation method, and the responses of the structure to the simulated wind excitation are obtained by numerically solving the nonlinear differential equations representing the model. The results of the numerical evaluation are compared with the corresponding field observations to validate the effectiveness of the model, and the validated model is used to investigate the effects of the structural damping and the turbulence intensity of wind on the characteristics of the coupled galloping of the tower.

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## 1. Introduction

Slender towers, such as lighting poles of various heights along highways, telecommunication towers and positive train control towers, have often been observed to exhibit large-amplitude wind-induced vibrations. Most of the problematic vibrations recorded in field monitoring campaigns have been identified to be either vortex-induced vibrations or buffeting responses of the tower to the excitation from wind turbulence (e.g., Chen et al., 2017; Connor et al., 2012; Zuo, 2008). However, many failures of slender towers have been found to be caused by wind-induced galloping (e.g., Caracoglia, 2007; Zuo et al., 2017), which is a type of aeroelastic oscillation at a frequency that is much lower than the frequency of vortex shedding from a structure. This is primarily because galloping oscillations can reach much larger amplitudes than can vortex-induced vibrations or buffeting responses, and many slender towers have cross-sections (such as those that are square in

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\* Corresponding author.
  *E-mail address:* delong.zuo@ttu.edu (D. Zuo).

shape) that render them susceptible to galloping. In addition, accretion of snow, ice or water rivulets on slender towers can also change the aerodynamic force coefficients of their cross-sections and potentially make these structures susceptible to galloping. For these reasons, it is necessary to assess the galloping oscillations of slender towers to ensure their safety and performance.

One type of galloping oscillation experienced by slender towers is of particular interest. It differs from the classical across-wind galloping in that it involves coupling between translations about the two principal axes of the towers (Solari and Pagnini, 1999; Zuo et al., 2017). Due to the coupling, the well-known Glauert-Den Hartog criterion (Den Hartog, 1932; Glauert, 1919) and the criteria proposed by Novak (1972), cannot be used to assess the onset of the galloping oscillation. For the same reason, the models for predicting the amplitude of the across-wind galloping, such as those that represent the self-excited forces by polynomials (e.g., Novak and Tanaka, 1974; Parkinson, 1984) or Fourier series (Richardson, 1988) of the velocity of the oscillation, are no longer applicable. A number of studies have proposed models for coupled galloping. Jones (1992) first formulated an eigenvalue problem that can be solved to yield a criterion for the onset of galloping that involves coupling between the along-wind and across-wind degrees of freedom. However, this formulation is restrictive in that it requires the interacting components of vibration to be at an identical frequency and the wind to be along one of the principal axes. Solari and Pagnini (1999) also formulated a model for the galloping of slender towers involving two orthogonal degrees of freedom about the principal axes. However, this model also only deals with the case when the mean wind direction is along a principal axis, and the focus of the study is also on the situation in which the frequencies of the two modes of vibration are identical.

In some other studies, models are developed to facilitate the evaluation of the onset of coupled galloping without the restrictions on the wind direction and the frequencies of the oscillation components. These include a model developed in a study of coupled galloping of tall buildings (Liang et al., 1993) and a model used to represent a perceived type of wind-induced dry stay cable vibration (Macdonald and Larose, 2008a,b). Although the latter model considers the effect of the Reynolds number on the wind loading, the general formulation of the model is also applicable when the Reynolds number is not relevant. A more recent study provided a comprehensive treatment of the galloping problem (Nikitas and Macdonald, 2014). It particularly highlighted the difference between across-wind galloping and galloping involving coupling between two translating degrees of freedom, as well as the effects of factors such as the frequency detuning of the participating modes on the onset and characteristics of coupled translational galloping. In another recent study, Zuo et al. (2017) presented the formulation of a model that can be used to assess the onset of coupled galloping of slender towers in boundary layer flows. The effectiveness of this model was illustrated through a comparison between the results from an evaluation of the onset condition of coupled galloping of a slender tower based on this model and the corresponding onset conditions observed from full-scale measurements of the tower.

However, all the models for the evaluation of the onset of coupled galloping resulted from simplification of the wind–structure interaction through linearization that is valid only when the oscillation amplitudes are small. For this reason, these models cannot be used to assess the characteristics of large-amplitude coupled galloping. Although a few nonlinear models have been developed to address this issue, none of those have been effectively applied in the evaluation of the coupled galloping of full-scale slender structures in boundary layer flows. For example, Li et al. (1998) formulated nonlinear equations of coupled galloping motion of slender structures in turbulent flows. However, in an illustrative application, this study unrealistically assumed that the turbulence is fully correlated along the height of the structure. In another study, Raeesi et al. (2014) derived the nonlinear equations of galloping motion of an inclined circular cylinder and used these equations as a basis to numerically assess the galloping response of the cylinder to the excitation from turbulent winds. Although the effects of the Reynolds number are included in the equations, a reduced version of this model can be readily applied to the cases in which the Reynolds number effect is trivial. However, the equations formulated in this study are only for a two-degree-of-freedom cylinder section in uniform flows and cannot be used to model the coupled galloping of a full-scale elastic tower in sheared boundary layer flows.

This paper presents the formulation of a set of nonlinear differential equations in state-space form as the representation of a model of coupled galloping of slender towers. It follows the general derivation used in Zuo et al. (2017), but does not linearize the fluid–structure interaction based on the assumption of small vibration amplitudes. Consequently, the model can be used to assess the characteristics of large-amplitude coupled galloping instead of only the onset of this type of oscillation. For illustration purposes, the model is used as a basis to numerically evaluate the coupled galloping of a full-scale tower in simulated boundary layer flows. Results of the numerical evaluation are compared with the observations from a previous field study that was conducted to monitor the wind-induced vibration of this tower (Zuo et al., 2017). In addition, the model is also used to study the effects of structural damping and wind turbulence on the major characteristics of the coupled galloping of the tower.

## 2. Analytical formulation of equations of motion

Fig. 1, which has been presented before in Zuo et al. (2017), schematically depicts the wind components, the resultant wind forces acting on a section of unit length of a slender structure and the translations of the section about its principal axes. The torsional response of the structure is neglected because it is insignificant for this type of structure (Solari and Pagnini, 1999). In this graph, $\overline{U}$, $u$ and $v$ are the mean along-wind speed of the wind and the along-wind and cross-wind components of the turbulence, respectively. $\overline{r}_x$ and $\overline{r}_y$ are the mean displacements of the section in the directions of the principal axes,