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Thermal boundary conditions for convective heat transfer of dilute gases in slip flow regime

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ABSTRACT

For gaseous flows in the slip flow regime, the power of the viscous stress at the wall is not zero. From the fluid domain point of view, it is a sink term, or a lost heat flux, that must be taken into account. It must be added to the diffusive heat flux in the fluid to appropriately model the heat flux transmitted from the fluid to the wall and the temperature field. The present technical note aims at theoretically establishing the appropriate thermal conditions in a general context: both thick and thin walls are considered as well as for imposed temperature (H1), imposed heat flux (H2) and convective heat transfer (H3) at the wall. Recent validations of this model, resulting from experimental and numerical comparisons of the convective heat transfer at the walls, are briefly discussed.

1. Introduction

In the last decades, the research activities about convective heat transfer in microdevices have rapidly been growing due to the considerable development of engineering applications. The Knudsen number, $Kn = \lambda/L$, defined as the ratio of the gas molecular mean-free-path, λ, to a characteristic length scale, L, such as the hydraulic diameter of a duct, allows a measure of the validity of the continuum approach and a classification of the gas flow regimes [1,2]. For $0.001 < Kn < 0.1$, the flow regime is called the slip-flow regime: the continuum assumption is still valid in the flow core but slip conditions, i.e. velocity slip, temperature jump and thermal creep, must be considered at the solid boundaries of the flow domain, to model the presence of the Knudsen layer (the very thin layer in a thermodynamical non-equilibrium state close to the solid boundary).

The flow field solution, described by a system of non-linear partial differential equations (conservation equations resulting from the Newton law, the first law of thermodynamics and the equation of state), depends on initial and boundary conditions. The expressions of the velocity-slip and thermal jump at solid walls for weakly rarefied flows have been established for a long time, by Maxwell [3] and Smoluchowski [4] and discussed since then by many authors. These boundary conditions that allow computing the fluid velocity and temperature accounting for the Knudsen layer are commonly used (at first or second order in Kn) in convective heat transfer modeling [5,6]. At this stage, there are not major controversies in the heat transfer literature.

However, when $Kn > 0.001$, in particular for gas flows in microdevices, some changes need to be brought to the expression of the heat flux transmitted through the wall: due to the slip velocity in the Knudsen layer, the power of the viscous stress is not zero at the wall and it must be taken into account in the thermal boundary condition to satisfy the heat flux consevation. This was first introduced by Maslen [7] and discussed by Sparrow and Lin [8]. Since then, only a few authors have taken into account the power of the viscous stress at the wall in their analysis of heat transfer [9–12]. So, as most of the authors neglected this contribution, it was eventually forgotten. Then Hong and Asako [13] reiterated its importance in 2010. However, it appears that the heat transfer community still goes on ignoring this boundary condition or uses it erroneously by considering that the imposed heat flux at the wall only balances the diffusive flux transmitted to the fluid. It can be thus underlined that most of the papers published in the archival literature on forced convective heat transfer in microdevices have provided erroneous values of the wall heat flux (or Nusselt number) or temperature field due to ill prescribed thermal boundary conditions.

The present technical note specifically focuses on the appropriate thermal boundary conditions for flows in microdevices in the slip regime and for first order slip models. After the work by Maslen (1958) [7] and Sparrow and Lin (1962) [8], the technical notes by Hadjiconstantinou (2003) [11],[1] and Hong and Asako (2010) [13] and the

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[1] Note that q_0 in Eq. (5) of [11] is the "thermal" heat flux, that is the diffusive part of the heat flux exchanged between the wall and the fluid, and not the imposed heat flux as considered by some authors [14], which led them to ill-interpretations of the boundary conditions.

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experimental and numerical verifications/validations by Shih et al. (2001) [9] for adiabatic walls, by Miyamoto et al. (2003) [10] for isoflux walls and Nicolas et al. (2017, 2018) [15,16] for isothermal walls, we demonstrate here that the power of the viscous stress at the wall must be included in the total heat flux transmitted by the fluid to the wall as soon as a slipping flow occurs at the wall.

A general demonstration and expressions of the right boundary conditions are provided. These BC's are established both for thick and thin walls, and for the H1, H2 and H3 boundary condition types. Finally, we briefly remind the published validations of the present formulation for parallel-plate channels submitted to H1 or H2 boundary conditions [9,10,15,16].

2. General expressions of the thermal conditions on a wall/slipping gas interface

In this paper, we only focus on the thermal boundary conditions. For the governing equations and the dynamical boundary conditions appropriate to model flows of compressible dilute gases in the slip flow regime, one can refer to our previous paper [16].

Let us first consider a gas flow domain of volume Ω_g with closed surface $\Gamma_g \cup \Gamma_i$ ($\Gamma_g \cap \Gamma_i = \oslash$), in contact along an interface Γ_i with a solid wall of thickness e, volume Ω_w, and closed surface $\Gamma_w \cup \Gamma_i = \Gamma_{w-} \cup \Gamma_{w,e} \cup \Gamma_{w+} \cup \Gamma_i$ ($\Gamma_{w-} \cap \Gamma_{w,e} \cap \Gamma_{w+} \cap \Gamma_i = \oslash$) (Fig. 1). We are interested in writting the thermal conditions at the interface Γ_i between the gas flow domain and the solid wall. To that aim, the energy equations in the gas and the solid are first integrated, for a steady problem and without source or sink terms for the sake of simplicity. Note that, even though the drawing in Fig. 1 is two-dimensional and the wall is flat, the present demonstration and formulations are general and can be applied to any three-dimensional geometry of the gas flow domain and solid wall.

By noting h the enthalpy and $e_c = \vec{v}^2/2$ the kinetic energy per mass unit of the gas, the steady conservation equation of the total energy of the gas writes (cf. page 341 in Ref. [17]):

$$\nabla.\left(\rho\vec{v}h + \rho\vec{v}e_c - k_g\nabla T_g - \bar{\bar{\tau}}.\vec{v}\right) = 0 \tag{1}$$

where the viscous stress tensor $\bar{\bar{\tau}}$ is defined for a Newtonian-Stokes fluid by:

$$\bar{\bar{\tau}} = \mu(\nabla\vec{v} + \nabla\vec{v}^{t}) - \frac{2}{3}\mu\nabla.\vec{v}\,\bar{\bar{I}} \tag{2}$$

and where $\vec{v}$, T_g, ρ, k_g and μ are the velocity, temperature, density, thermal conductivity and dynamical viscosity of the gas. By integrating Eq. (1) on Ω_g of closed surface $\Gamma_g \cup \Gamma_i$ (Fig. 1), considering the wall as impermeable ($\vec{v}.\vec{n} = 0$ on Γ_i) and using Gauss'theorem, we get:

$$\int_{\Gamma_g}\left(\rho(h+e_c)\vec{v} - k_g\nabla T_g - \bar{\bar{\tau}}.\vec{v}\right).\vec{n}_g d\Gamma_g + \underbrace{\int_{\Gamma_i}\underbrace{\left(-k_g\nabla T_g - \bar{\bar{\tau}}.\vec{v}\right)}_{=\vec{q}_i}.\vec{n}_i d\Gamma_i}_{=Q_i} = 0 \tag{3}$$

where $\vec{n}_g$ is the inward normal unit vector on Γ_g, directed from the wall to the gas and denoted $\vec{n}_i$ on Γ_i. In Eq. (3), the first integral term is the sum of the enthalpy and kinetic energy convective fluxes, the conductive heat flux and the power of the viscous stress through the fluid boundary Γ_g; the second integral term represents the total heat flux, Q_i $[W]$, transmitted by the fluid to the wall through the interface Γ_i. By definition Q_i is the integral on Γ_i of the total local heat flux density, $q_i = \vec{q}_i.\vec{n}_i$ $[W/m^2]$, transmitted by the fluid through Γ_i. Thus q_i is the sum of the diffusive heat flux density and the power of the viscous stress at the wall:

$$q_i = \left(-k_{g,i}\nabla T_{g,i} - (\bar{\bar{\tau}}.\vec{v})_{g,i}\right).\vec{n}_i \tag{4}$$

where the subscript "g, i" denotes quantities on the gas side of the gas/wall interface (slip-related quantities associated with the gas molecules in contact with the wall).

In the solid wall, with the used assumptions, the energy equation simply writes:

$$\nabla.(-k_w\nabla T_w) = 0 \tag{5}$$

with k_w and T_w the thermal conductivity and temperature of the solid wall. By integrating Eq. (5) on Ω_w of closed surface $\Gamma_w \cup \Gamma_i$ with $\Gamma_w = \Gamma_{w-} \cup \Gamma_{w,e} \cup \Gamma_{w+}$ (Fig. 1) and using Gauss's theorem, we get:

$$\int_{\Gamma_w}(-k_w\nabla T_w).\vec{n}_w d\Gamma_w + \underbrace{\int_{\Gamma_i}\underbrace{(-k_w\nabla T_w)}_{=\vec{q}_i}.\vec{n}_i d\Gamma_i}_{=Q_i} = 0 \tag{6}$$

where $\vec{n}_w$ is the outward normal unit vector on Γ_w. Here the first integral term is the conductive heat flux through the solid boundary Γ_w and the second integral is the conductive heat flux through Γ_i. This second term thus represents the total heat flux transmitted by the solid through the interface. It is therefore also equal to Q_i since, from the continuum mechanics laws, the heat flux is conserved through a zero volume interface without source term. Furthermore, by identification, the heat flux density transmitted locally by the solid wall through the interface is:

$$q_i = (-k_{w,i}\nabla T_{w,i}).\vec{n}_i \tag{7}$$

where the subscript "w, i" denotes quantities on the wall side of the interface. Thus, from Eqs. (4) and (7), the conservation of the heat flux density, q_i, transmitted locally, through an infinitesimal gas/wall interface $d\Gamma_i$, writes:

$$(-k_{w,i}\nabla T_{w,i}).\vec{n}_i = \left(-k_{g,i}\nabla T_{g,i} - (\bar{\bar{\tau}}.\vec{v})_{g,i}\right).\vec{n}_i \tag{8}$$

When solving the energy equations (1) and (5) to compute the two unknown temperature fields, T_g and T_w, for the gas and the wall, two interface conditions are necessary. The first one is given by Eq. (8): it expresses the total heat flux conservation through the interface. The second one expresses the temperature jump between the local interfacial temperatures of the solid wall, $T_{w,i}$, and the gas, $T_{g,i}$. It reads [6,18]:

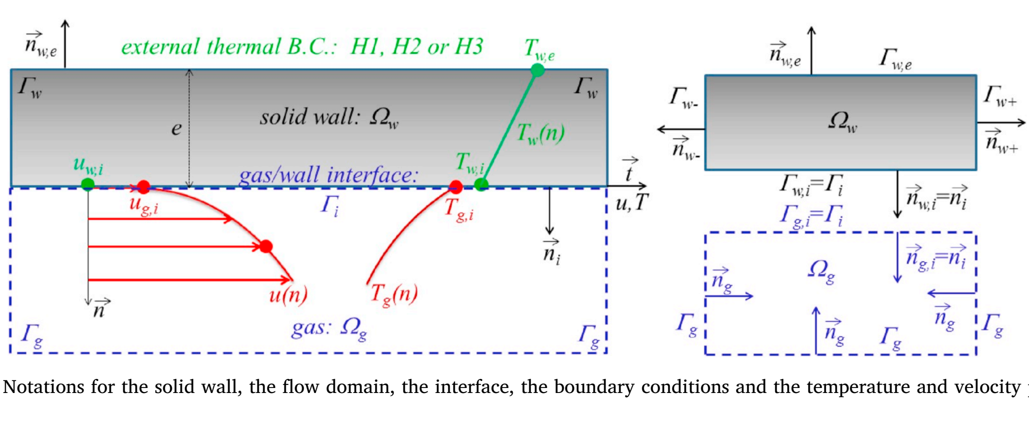

Fig. 1. Notations for the solid wall, the flow domain, the interface, the boundary conditions and the temperature and velocity profiles.