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Obtaining some constraints on the early universe based on WMAP-9 and Planck by using DECIGO and BBO detectors

Basem Ghayour

School of Physics, University of Hyderabad, Hyderabad-500 046 India

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ABSTRACT

The evolution stages of the universe such as: inflation, reheating, radiation, matter and acceleration can affect on the shape of the spectrum of relic gravitational waves. As well known, at the end of inflation, the scalar field ϕ oscillates quickly around some point where potential $V(\phi) = \lambda\phi^n$ has a minimum. The potential V causes that the scale factor S obtains unusual growth during inflation, as the amount of this growth is given by e^N, where N is the e-folding number. On the other hand the behaviour of the inflation and reheating stages are often known as power law expansion like $S(\eta) \propto \eta^{1+\beta}, S(\eta) \propto \eta^{1+\beta_s}$ respectively. The η is conformal time and β, β_s constrained on the $1+\beta < 0$ and $1+\beta_s > 0$. The waves being used to determine the reheating temperature in the range $(10^6–10^9)$ GeV based on the BBO and DECIGO detectors. Hence we found constraints on the parameters N, β, β_s and n corresponding to the range of reheating temperature, WMAP-9 and Planck. As these constraints give us valuable information about the manner of the evolution of universe and the waves during inflation and reheating stages.

1. Introduction

The relic gravitational waves (RGWs) contain lot of information about the very early universe. There are some stages for the evolution of universe from the very early time until now like: inflation, reheating, radiation, matter dominate and acceleration. Each of these stages can affect on the shape of the spectrum of the RGWs. As by studying these effects we can understand about the history and the manner of the evolution of universe. We assume the slow-roll inflation which is in good agreement with the cosmic microwave background observations. The scalar field ϕ of general case of potential $V(\phi) = \lambda\phi^n$ [1] has a minimum during the end of inflation where λ is a constant and n is a free parameter. The potential V causes that the scale factor S obtains unusual growth during inflation, as the amount of this growth is given by e^N, where N is the e-folding number. Also the reheating stage was essential for the nucleosynthesis process since the inflation brought temperature of the universe below for the requirement of thermonuclear reactions. We know that frequency of the spectrum varies due to the reheating temperature (T_{rh}) in the reheating stage [2]. This T_{rh} must be larger than a few MeV [3], for the creation light elements, but less than the energy scale at the end of inflation as $T_{rh} < 10^{16}$ GeV [4]. Especially T_{rh} may be determined by future detectors of the waves such as DECIGO and/or BBO if it is around $(10^6–10^9)$ GeV [5]. We prefer to call this range as 'especial range' in this work. On the other hand the behaviour of the inflation and reheating stages are often known as power law expansion like $S(\eta) \propto \eta^{1+\beta}, S(\eta) \propto \eta^{1+\beta_s}$ respectively. The η is conformal time and β, β_s constrained on the $1+\beta < 0$ and $1+\beta_s > 0$ [6], [7]. As β and β_s play main role in the spectrum of the RGWs through the general range of frequency $\sim (10^{-20}–10^{11})$ Hz. Therefore based on the description as given above, inflation and reheating stages contain valuable information about the early universe.

Thus in order to obtain the information, we are interested to find some constraints on the N, n, β, β_s based on WMAP-9 and Planck by using the estimated especial range of T_{rh} from DECIGO and BBO. We use the units $c = \hbar = k_B = 1$.

2. Evolution of gravitational waves

We can write the perturbed metric for a homogeneous isotropic flat Friedmann–Robertson–Walker universe as follows

$$ds^2 = S^2(\eta)(d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j), \tag{1}$$

where $S(\eta)$, η, δ, h_{ij} are cosmological scale factor, conformal time, Kronecker delta symbol and metric perturbations respectively. The h_{ij} contain only the pure gravitational waves and are transverse-traceless i.e; $\nabla_i h^{ij} = 0, \delta^{ij} h_{ij} = 0$.

In this work, we are considering the shape of the spectrum of RGWs generated by the expanding space time background. Hence the perturbed matter source is not taken into account. One can

E-mail address: ba.ghayour@gmail.com

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describes the RGWs with the linearised field equation that given by

$$\nabla_\mu\left(\sqrt{-g}\,\nabla^\mu h_{ij}(\mathbf{x},\eta)\right)=0. \quad (2)$$

The tensor perturbations have two independent physical degrees of freedom like h^+ and $h^\times$ and called polarisation modes. To compute the spectrum of RGWs $h(\mathbf{x},\eta)$, we express h^+ and $h^\times$ in terms of the creation ($a\dagger$) and annihilation (a) operators,

$$h_{ij}(\mathbf{x},\eta)=\frac{\sqrt{16\pi}\,l_{pl}}{S(\eta)}\sum_{\mathbf{p}}\int\frac{d^3k}{(2\pi)^{3/2}}\epsilon_{ij}^{\mathbf{p}}(\mathbf{k})\frac{1}{\sqrt{2k}}\left[a_{\mathbf{k}}^{\mathbf{p}}h_{\mathbf{k}}^{\mathbf{p}}(\eta)e^{i\mathbf{k}.\mathbf{x}}+a_{\mathbf{k}}^{\dagger\,\mathbf{p}}h_{\mathbf{k}}^{*\,\mathbf{p}}(\eta)e^{-i\mathbf{k}.\mathbf{x}}\right], \quad (3)$$

where $l_{pl}=\sqrt{G}$ is the Planck's length, $\mathbf{k}$ is the comoving wave number, $k=|\mathbf{k}|$, and $\mathbf{p}=+,\times$ are polarisation modes. The polarisation tensor $\epsilon_{ij}^{\mathbf{p}}(\mathbf{k})$ is symmetric and transverse-traceless $k^i\epsilon_{ij}^{\mathbf{p}}(\mathbf{k})=0,\delta^{ij}\epsilon_{ij}^{\mathbf{p}}(\mathbf{k})=0$ and satisfy the conditions $\epsilon^{ij\mathbf{p}}(\mathbf{k})\epsilon_{ij}^{\mathbf{p}'}(\mathbf{k})=2\delta_{\mathbf{pp}'}$ and $\epsilon_{ij}^{\mathbf{p}}(-\mathbf{k})=\epsilon_{ij}^{\mathbf{p}}(\mathbf{k})$. The a and $a^\dagger$ satisfy $[a_{\mathbf{k}}^{\mathbf{p}},a_{\mathbf{k}'}^{\dagger\,\mathbf{p}'}]=\delta_{\mathbf{pp}'}\delta^3(\mathbf{k}-\mathbf{k}')$ and the initial vacuum state is defined as $a_{\mathbf{k}}^{\mathbf{p}}|0\rangle=0$ for each $\mathbf{k}$ and $\mathbf{p}$. For a fixed wave number $\mathbf{k}$ and a fixed polarisation state $\mathbf{p}$, Eq. (2) gives

$$h_k''+2\frac{S'}{S}h_k'+k^2h_k=0, \quad (4)$$

where prime is derivative with respect to the conformal time. We consider $h_k(\eta)$ without the polarisation index, since the polarisation states are same.

Then, we rescale $h_k(\eta)$ by taking $h_k(\eta)=f_k(\eta)/S(\eta)$, where the mode functions $f_k(\eta)$ obey the minimally coupled Klein–Gordon equation

$$f_k''+\left(k^2-\frac{S''}{S}\right)f_k=0. \quad (5)$$

The general solution of this equation is given in Appendix A. On the other hand the power spectrum of gravitational waves is defined as

$$\int_0^\infty h^2(k,\eta)\frac{dk}{k}=\langle 0|h^{ij}(\mathbf{x},\eta)h_{ij}(\mathbf{x},\eta)|0\rangle. \quad (6)$$

Substituting Eq. (3) in Eq. (6) and taking the contribution from each polarisation is same, we get

$$h(k,\eta)=\frac{4l_{pl}}{\sqrt{\pi}}k|h(\eta)|, \quad (7)$$

thus once the mode function $h(\eta)$ is known, the spectrum $h(k,\eta)$ follows.

The spectrum at the present time $h(k,\eta_0)$ can be obtained, provided the initial spectrum is specified. The initial condition is taken to be during the inflation. The wave with wave number k crossed over the horizon at a time η_i, when the wavelength $\lambda_i/2\pi=1/H(\eta_i)=S(\eta_i)/k$ [8]. Now we choose the initial condition of the mode function h_k as $|h_k(\eta_i)|=1/S(\eta_i)$. The initial amplitude of the power spectrum is

$$h(k,\eta_i)=8\sqrt{\pi}\frac{l_{pl}}{\lambda_i}. \quad (8)$$

With $\lambda_i/2\pi=1/H(\eta_i)$ it becomes

$$\frac{S'(\eta_i)}{S(\eta_i)}=k. \quad (9)$$

Therefore initial amplitude of the spectrum is given by

$$h(k,\eta_i)=A\left(\frac{k}{k_H}\right)^{2+\beta}, \quad (10)$$

where the constant A can be determined by quantum normalisation [7]:

$$A=8\sqrt{\pi}\frac{l_{pl}}{l_0}. \quad (11)$$

Thus the amplitude of the spectrum for different ranges are given as follows [6, 7, 9, 10].

$$h(k,\eta_0)=A\left(\frac{k}{k_H}\right)^{2+\beta},\quad k\le k_E, \quad (12)$$

$$h(k,\eta_0)=A\left(\frac{k}{k_H}\right)^{\beta-\gamma}(1+z_E)^{\frac{-2-\gamma}{\gamma}},\quad k_E\le k\le k_H, \quad (13)$$

$$h(k,\eta_0)=A\left(\frac{k}{k_H}\right)^{\beta}(1+z_E)^{\frac{-2-\gamma}{\gamma}},\quad k_H\le k\le k_2, \quad (14)$$

$$h(k,\eta_0)=A\left(\frac{k}{k_H}\right)^{1+\beta}\left(\frac{k_H}{k_2}\right)(1+z_E)^{\frac{-2-\gamma}{\gamma}},\quad k_2\le k\le k_s, \quad (15)$$

$$h(k,\eta_0)=A\left(\frac{k}{k_H}\right)^{1+\beta-\beta_s}\left(\frac{k_s}{k_H}\right)^{\beta_s}\left(\frac{k_H}{k_2}\right)(1+z_E)^{\frac{-2-\gamma}{\gamma}},\quad k_s\le k\le k_1, \quad (16)$$

where $k_E=\frac{k_H}{1+z_E}$, see Appendix A for more details.

We take the value of redshift $z_E\sim 0.3$ and $\gamma\simeq 1.05$ [11] for $\Omega_\Lambda=0.692$ from Planck 2015 [12]. By taking $\lambda/2\pi=1/H$ [8] and ν as frequency, we can obtain $\nu_E=3\times10^{-19}$ Hz, $\nu_H=3.6\times10^{-19}$ Hz, $\nu_2=1.4\times10^{-17}$ Hz, $\nu_s=1.5\times10^9$ Hz.

The spectral energy density parameter $\Omega_g(\nu)$ of gravitational waves is defined through the relation $\rho_g/\rho_c=\int\Omega_g(\nu)\frac{d\nu}{\nu}$, where ρ_g, ρ_c are the energy density of the gravitational waves and critical energy density respectively. Therefore we have [7]

$$\Omega_g(\nu)=\frac{\pi^2}{3}h^2(k,\eta_0)\left(\frac{\nu}{\nu_H}\right)^2. \quad (17)$$

We assume that the space time is spatially flat $K=0$ with $\Omega=1$, then the fraction density of RGWs must be less than unity, $\rho_g/\rho_c<1$. In order to ρ_g/ρ_c dose not exceed the level of 10^{-5}, one gets $\Omega_g(\nu_1)\simeq10^{-6}$ in Eq. (17), therefore we get $\nu_1\simeq4\times10^{10}$ Hz for the acceleration stage of universe [2].

One can get the constant A without scalar running as follows [8]

$$A=\frac{\Delta_R(k_0)r^{1/2}}{(1+z_E)^{\frac{-2-\gamma}{\gamma}}}\left(\frac{\nu_H}{\nu_0}\right)^\beta, \quad (18)$$

where $\Delta_R^2(k_0)$ is the power spectrum of the curvature perturbation evaluated at the pivot wave number $k_0^p=k_0/a(\eta_0)=0.002$ Mpc^{-1} [12] with corresponding physical frequency $\nu_0\simeq4.9\times10^{-19}$ Hz. The $\Delta_R^2(k_0)=(2.464\pm0.072\times10^{-9})$ is given by WMAP9 + eCMB + BAO + H_0 [13]. The tensor to scalar ratio $r<0.11(95\%CL)$ is based on Planck measurement [14]. We take $r\simeq0.1$ and also value of redshift $z_E=0.3$ for TT, TE, EE+lowP+lensing contribution in this work [12].

3. The estimated constraints on N, n, β and β_s

At the end of inflation, the scalar field ϕ oscillates quickly around some point where potential $V(\phi)=\lambda\phi^n$ has a minimum. The scalar power spectrum by tacking in account slow-roll inflation is as follows:

$$P_s(k)=\frac{1}{\pi\epsilon}\left(\frac{H}{m_{pl}}\right)^2\left(\frac{k}{SH}\right)^{n_s-1}, \quad (19)$$