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Shape inverse problem for Stokes-Brinkmann equations *

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Abstract In this paper, we propose an imaging technique for the detection of porous inclusions in a stationary flow governed by Stokes-Brinkmann equations. We introduce the velocity method to perform the shape deformation, and derive the structure of shape gradient for the cost functional based on the continuous adjoint method and the function space parametrization technique. Moreover, we present a gradient-type algorithm to the shape inverse problem. The numerical results demonstrate the proposed algorithm is feasible and effective for the quite high Reynolds numbers problems.

Keywords Stokes-Brinkmann equations; shape inverse problem; shape gradient; continuous adjoint method; function space parametrization technique

1 Introduction

The inverse problem that we consider in this paper is to image the penetrable inclusion S modeled by the Stokes-Brinkmann equations in a bounded Lipschitz domain $\Omega \subset \mathbb{R}^N, N = 2$ or 3. These equations describe a fluid in steady-state in terms of the velocity field $\boldsymbol{u} = (u_1, ..., u_N)^T : \Omega \to \mathbb{R}^N$ and pressure $p : \Omega \to \mathbb{R}$, and also involve the kinematic viscosity ν and the matrix-valued function $M : \Omega \to \mathbb{R}^{N\times N}$,

$$\begin{cases} -\operatorname{div}\sigma(\boldsymbol{u}, p) + M\boldsymbol{u} = 0 & \text{in} \quad \Omega, \\ \operatorname{div}\boldsymbol{u} = 0 & \text{in} \quad \Omega, \\ \boldsymbol{u} = 0 & \text{on} \quad \Gamma_s \cup \Gamma_w, \\ \boldsymbol{u} = \boldsymbol{g} & \text{on} \quad \Gamma_{in}, \\ \sigma(\boldsymbol{u}, p) \cdot \boldsymbol{n} = 0 & \text{on} \quad \Gamma_{out}, \end{cases} \tag{1.1}$$

where $\sigma(\boldsymbol{u}, p)$ is the stress tensor defined by $\sigma(\boldsymbol{u}, p) := -p\,\mathrm{I} + 2\nu\varepsilon(\boldsymbol{u})$ with the rate of deformation tensor $\varepsilon(\boldsymbol{u}) := (\mathrm{D}\boldsymbol{u} + {}^*\mathrm{D}\boldsymbol{u})/2$, and ${}^*\mathrm{D}\boldsymbol{u}$ denotes the transpose of the matrix $\mathrm{D}\boldsymbol{u}$. The boundary of Ω is smooth, and consists of four parts. Γ_{in} is the inflow boundary, Γ_{out} denotes the outflow boundary, Γ_w represents the boundary corresponding to the fluid wall, and Γ_s is the boundary of inclusion S.

The purpose of this paper is to recover the boundary Γ_s which minimizes a tracking cost functional J,

$$\min_{\Omega\in\mathcal{O}} J(\Omega) = \frac{1}{2}\int_\Omega |\boldsymbol{u} - \boldsymbol{u}_d|^2 \,\mathrm{d}x, \tag{1.2}$$

where $\boldsymbol{u}_d$ is the measured velocity on an observation set. Let the boundary $\Gamma_D := \Gamma_{in} \cup \Gamma_w \cup \Gamma_{out}$ be fixed in the shape identification problem, and the admissible set can be given by $\mathcal{O} := \{\Omega \subset \mathbb{R}^2 : \ \Gamma_D \text{ is fixed}, \ \int_\Omega 1 \,\mathrm{d}x = \text{constant}\}$.

The Stokes-Brinkmann equations are important for modeling flow through porous and partially porous media. The applications include for instance the modeling of liquids or gas through the ground. The problem of detecting or identifying the inclusions in flows could be applied to nondestructive testing problems.

The shape identification problem has been a challenging task for a long time. In general, it refers to very large computational costs. Besides the numerical approximation of partial differential equations, it also requires a suitable approach for representing and deforming the shape of the underlying geometry. As far as we know, several methods have been proposed

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