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An improved analytical solution for MPP parameters of photovoltaic cells

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A B S T R A C T

Aimed at the puzzle that the maximum power point (MPP) parameters of photovoltaic (PV) cells cannot be readily solved, this paper starts with a detailed theoretical analysis on the analytical solutions for MPP parameters under the ideal circuit model on the grounds of the mean value theorem, and then the exact analytical expressions for MPP parameters are deduced by referring to the general circuit model of PV cells. Of the analytical expressions deduced in this paper, the variables are directly correlated, and their solutions are handy and demanding no iterative algorithms. Next, the effects of cell temperature and solar irradiance on each component of the analytical expressions are dissected in detail, with the minor components ignored, so as to uncover a linear relationship among MPP parameters, cell temperature and solar irradiance. Eventually, approximate linear estimation formulas for MPP parameters are obtained. The validity of the two deduced formulas is then verified by various simulation experiments and field measurement experiments, both of which prove that the acquired parameters have a high accuracy for practical applications. Moreover, the formulas are open for selection or further simplification in accordance with different requirements on computation loads of algorithms or varying needs for parameter accuracy.

1. Introduction

Nowadays, as fossil fuels are running short, countries all over the world are painstakingly geared to develop a variety of renewable energy. Being clean, inexhaustible, and of highest power density, solar energy has become the world's third largest renewable resource of electric power (Jordehi, 2016; Bai et al., 2014). However, photovoltaic (PV) cells are an unstable power supply, because their output power is subject to solar irradiance, cell temperature and the size of load resistance among other factors. Therefore, maximum power point (MPP) parameters, particularly the interactions among MPP voltage, MPP current, solar irradiance and cell temperature, need to be identified in the case of power generating systems for PV cells.

Currently, the electrical parameters of PV cells are mainly solved by mathematical modeling via softwares like Matlab, Maple and Mathematica: namely, single-diode modeling (Villalva et al., 2009; Shongwe and Hanif, 2015), dual-diode modeling (Gow and Manning, 1999) and triple-diode modeling (Nishioka et al., 2007). These mathematical models are all based on the inner circuit structure of PV cells, which can visualize the effect of photo-electric conversion. However, they fail to build a linear and unambiguous connection between voltage and current, and involve the implicit transcendental equation as well. The solution of this equation is usually achieved via the Newton iteration method and other related algorithms, the solving process of which is cumbersome and brings convergence difficulty (Batzelis et al., 2015; Lun et al., 2013).

Subsequently, scholars have made improvements on these mathematical models related to PV cells or introduced intelligent algorithms to simplify the solving process or increase the accuracy of solutions. Ma et al. (2014) has established a new theoretical model by citing electrical parameters provided by manufacturers, which achieves a sound compromise between accuracy and simplicity, although its solution requires the Levenberg-Marquardt (LM) algorithm and the Gauss-Newton algorithm. Saetre et al. (2011) use a simple mathematical equation to represent the empirical *I-V* curve of PV cells, whereas this equation can only be used in a simple way, due to the fact that it is not derived from a physical theory. With a rigorous theoretical derivation, Batzelis et al. (2015) obtain direct solution formulas for MPP parameters under ideal conditions, non-ideal conditions and partial shading conditions, on the footing of the Lambert W function. However, this method still needs to undergo a complex numerical solution for the Lambert W function via a mathematical software. In addition, some artificial intelligence algorithms have also been introduced to solve the electrical parameters of PV cells, such as the genetic algorithm (Zagrouba et al., 2010), the pattern search algorithm (Alhajri et al., 2012), and the artificial bee swarm optimization algorithm (Askarzadeh and Rezazadeh, 2013), all

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of which can achieve a solid accuracy in parameters, in spite of such demerits as colossal computing and convergence difficulty.

Other analytical methods aimed to solve MPP parameters have also been proposed to overcome the drawbacks of numerical methods. Embracing the mean value theorem, Rodriguez and Amaratunga (2007) discover, by deduction, that the derivative of MPP current versus MPP voltage approximates to the opposite number of the ratio of short-circuit current to open-circuit voltage, thereby obtaining the analytical expressions for MPP parameters. By applying the same assumption, Saloux et al. (2011) deduce the explicit formulas for MPP parameters from the ideal single-diode model. Proceeding from the method proposed by Saloux et al. (2011), Fernandes et al. (2014) propose the complex but analytical mathematical expressions for MPP load resistance and MPP current by weighing the impacts of shunt resistance, and series resistance. However, computation errors of MPP parameters do emerge, either with the assumption on the derivative of MPP current versus MPP voltage, or in the application of the ideal single-diode model. Radziemska and Klugmann (2006) manage to obtain a linear relationship between electrical parameters of PV modules, cell temperature and solar irradiance through experiments, but with regard to different PV modules, either the temperature coefficient or the radiation-rate coefficient of their electrical parameters could only be obtained through complicated experiments. Some other research papers have directly given various linear or approximate linear estimation formulas about the MPP parameters of PV cells, without providing their corresponding process of theoretical derivation (Jordehi, 2016; Lun et al., 2013; Khezzar et al., 2014; Rajasekar et al., 2013).

This paper, however, first deduces analytical formulas for solving MPP parameters under the ideal circuit model, on the grounds of the mean value theorem, and by theoretical derivation. Next, exact analytical expressions for MPP parameters are further deduced on the basis of the general circuit model, by taking into account the effects of both shunt resistance and series resistance on the MPP parameters. Thereafter, the effects of cell temperature and solar irradiance on each component of the analytical expressions are analyzed, with the minor components ignored, in order to establish an approximate linear relationship between MPP parameters and cell temperature as well as solar irradiance. Then approximate linear estimation formulas for MPP parameters are theoretically derived. Finally, based on polycrystalline silicon, monocrystalline silicon and thin-film PV cells, simulation experiments and field measurement experiments are carried out to verify the effectiveness of the two solving methods proposed in this paper.

2. Mathematical models of PV cells

Among numerous models, the single-diode circuit models can match PV cells in actual use with higher precision, and it has made a sound compromise between precision and simplicity (Toledo and Blanes, 2014; Villalva et al., 2009). Fig. 1 shows single-diode equivalent circuit models of PV cell, which are also adopted in this paper.

Fig. 1a shows the single-diode ideal circuit model of PV cell, which comprises a parallel connection between a photocurrent I_{ph} and a semiconductor diode D. I_{id} and V_{id} stand for the ideal output current and the ideal output voltage respectively, while $R_{L,id}$ represents the load resistor externally connected to the ideal circuit model. In addition, in order to obtain enough voltage, the actual module of PV cells assembles

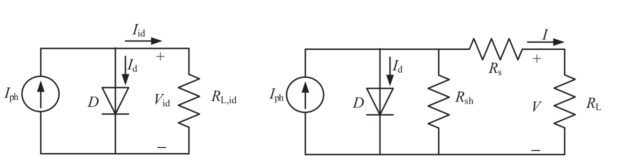

Fig. 1. Single-diode equivalent circuit models of PV cell: (a) ideal circuit model and (b) general circuit model.

N_s pieces of individual cell via series connection, since the voltage of each individual cell is only about 0.5 V (Tsai et al., 2008). Hence, the expression for the *I-V* characteristic curve of the ideal circuit model of PV cells is shown as follows:

$$I_{id} = I_{ph} - I_o\left[exp\left(\frac{qV_{id}}{nN_s kT}\right) - 1\right] \tag{1}$$

where I_o is the reverse saturation current of the diode, q being the charge amount of an electron (1.6×10^{-19} C), k being the Boltzmann constant (1.38×10^{-23} J/K), T being the cells temperature, and n being the ideality factor.

I_o is not only determined by the structure and material property of the PN junction, but also greatly influenced by temperature. Therefore, the equation for calculating I_o becomes a function about temperature, and it is generally assumed that I_o is proportional to the third power of temperature, as shown in Eq. (2) (Ma et al., 2014):

$$I_o = I_{o,r}\left(\frac{T}{T_r}\right)^3 exp\left[\frac{qE_g}{nk}\left(\frac{1}{T_r} - \frac{1}{T}\right)\right] \tag{2}$$

where E_g refers to the bandgap energy of semiconductor material, T_r being the reference temperature of PV cells (25 °C), and $I_{0,r}$ being the value of reverse saturation current under reference temperature. Based on $I_{0,r}$, Villalva et al. (2009) introduce two temperature coefficients, μ_I for short-circuit current and μ_V for open-circuit voltage. Then short-circuit current and open-circuit voltage is expressed as a linear function involving temperature, whereby another equation for I_o is obtained:

$$I_o = \frac{I_{ph,r} + \mu_I(T - T_r)}{exp\left[\frac{q(V_{oc,r} + \mu_V(T - T_r))}{nN_s kT}\right] - 1} \tag{3}$$

where $I_{ph,r}$ and $V_{oc,r}$ refer respectively to the short-circuit current and the open-circuit voltage under the standard test conditions (with solar irradiance at 1000 w/m^2, and the temperature kept at 25 °C). Eq. (3) is also used herein as the computing formula for I_o.

The ideality factor n of PV cells represents the ideal rating of the diode, mainly affecting the curvature of the *I-V* characteristic curve (Kim and Choi, 2010). The value of n depends on the material that is used to manufacture PV cells, and is also somewhat related to the temperature of the diode, which is usually within a numerical interval between 1 and 2. In this paper, it is taken as a constant, in terms of each kind of specific material.

Because the ideal circuit model cannot accurately reflect the true *I-V* characteristic curve of PV cells, a shunt resistance R_{sh} and a series resistance R_s are added to characterize the resistance property and the internal current loss of specific material, whereby a general circuit model is obtained. Fig. 1b illustrates a single-diode general circuit model for PV cells, with I as the output current, V as the output voltage, and R_L as the externally connected load resistance. The expression for the *I-V* characteristic curve of the general circuit model is as shown in Eq. (4):

$$I = I_{ph} - I_o\left\{exp\left[\frac{q}{nN_s kT}\left(V + IRs\right)\right] - 1\right\} - \frac{V + IR_s}{R_{sh}} \tag{4}$$

A large number of methods are accessible for the numerical estimation of R_{sh} and R_s, whereas their values are taken as constants for each specific kind of PV cells discussed in this paper, because both of them are assumed to have nothing to do with solar irradiance and cell temperature. The specific values of R_{sh} and R_s should be solved by applying the iterative algorithm proposed by Villalva et al. (2009) to obtain a numerical pair (R_s, R_{sh}) in the standard test conditions, thereupon to ensure that the peak of the mathematical *P-V* curve is consistent with the experimental one.

The presence of serial resistance R_s and shunt resistance R_{sh} will also change the curvature of the *I-V* characteristic curve, especially