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On the dynamics of imperfect shear deformable microplates

Hamed Farokhi[a], Mergen H. Ghayesh[b,*]

[a] Department of Mechanical and Construction Engineering, Northumbria University, Newcastle upon Tyne NE1 8ST, UK
[b] School of Mechanical Engineering, University of Adelaide, South Australia 5005, Australia

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ABSTRACT

This paper investigates the nonlinear forced dynamical behaviour of a geometrically imperfect viscoelastic shear-deformable microplate. The third-order shear deformation plate theory and the Kelvin–Voigt viscoelastic model are utilised in the framework of the modified version of the couple-stress theory to develop a model for the microsystem. The developed model is in the form of partial differential equations (PDEs) and accounts for geometric nonlinearities, damping nonlinearities, micro-scale size effects, and initial imperfection. Five coupled PDEs are derived for the five independent displacements and rotations. These PDEs are truncated to a set of nonlinearly coupled ordinary differential equations via application of a two-dimensional modal decomposition based on the Galerkin technique. The final set of equations consists of quadratic and cubic nonlinear terms for both damping and stiffness. An efficient numerical algorithm based on a continuation scheme is utilised to analyse the nonlinear forced vibration characteristics of such complicated system. The effects imperfection amplitude, damping nonlinearities, and micro-scale size on forced resonant vibration response are highlighted.

1. Introduction

Since the major mechanical parts of microelectromechanical systems (MEMS) (Farokhi & Ghayesh, 2018; Ghayesh, Farokhi, & Alici, 2016; Ghayesh, Farokhi, & Amabili, 2013; Li & Pan, 2015; Medina, Gilat, & Krylov, 2017; Mojahedi, 2017; Qi, Huang, Fu, Zhou, & Jiang, 2018) are commonly microbeams/microplates (Dehrouyeh-Semnani, BehboodiJouybari, & Dehrouyeh, 2016; Hosseini & Bahaadini, 2016; Mojahedi & Rahaeifard, 2016; Taati, 2016), a better understanding of the nonlinear forced vibrational characteristics of such systems is essential toward development of more efficient and reliable designs for MEMS. Microsystems and MEMS applications range from microscale sensors and resonators to micro energy harvesters. In such systems, the microsystem is usually excited in the nonlinear regime; one of the important factors affecting the oscillation amplitude is the presence of an initial geometric imperfection (Farokhi & Ghayesh, 2016; Farokhi & Ghayesh, 2017; Farokhi, Ghayesh, & Amabili, 2013; Ghayesh, Farokhi, & Gholipour, 2017). Another factor which significantly affects the numerical predictions is the nonlinearities associated with damping. In fact, it is shown that the dissipation is nonlinear in microresonators and that the internal energy dissipation affects the forced oscillation of microstructures (Zaitsev, Shtempluck, Buks, & Gottlieb, 2012). One efficient method of modelling the damping nonlinearities is to employ a material damping

* Corresponding author.
E-mail address: mergen.ghayesh@adelaide.edu.au (M.H. Ghayesh).

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mechanism, such as Kelvin–Voigt (Ghayesh, Farokhi, & Hussain, 2016), Maxwell, or Zener, in conjunction with geometric nonlinearities. In fact, since these damping mechanisms are strain dependent, the presence of geometric nonlinearities in the strain-displacement relation renders them nonlinear damping models. In the present study, the Kevin-Voigt material damping model is employed to account for nonlinearities associated with damping in the forced resonant vibrational characteristics of the initially imperfect shear deformable microplate.

Another factor of importance in modelling micro/nano structure is the micro-scale size effect (Attia & Abdel Rahman, 2018; Bahaadini, Saidi, & Hosseini, 2018; Ghayesh, Amabili, & Farokhi, 2013a, b; Ghayesh, Farokhi, & Amabili, 2013; Hadi, Nejad, & Hosseini, 2018; Khaniki, 2018; Li, Tang, & Hu, 2018). It has been shown experimentally (Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; McFarland & Colton, 2005) that at small-scales, the structural behaviour of the system is affected by the size and hence cannot be modelled properly through use of classical continuum theories (Bakhshi Khaniki & Hosseini-Hashemi, 2017; Farokhi & Ghayesh, 2015; Ghayesh, Farokhi, & Amabili, 2014; Kiani, 2016; Shahverdi & Barati, 2017; She, Yuan, Ren, & Xiao, 2017). To address this problem, the higher-order modified couple stress theory (MCST) (Ghayesh & Farokhi, 2015) is employed in this study to highlight the micro-scale size effects: Ghayesh (2018b); Ghayesh (2018c); Ghayesh (2018d).

A review of the literature shows that there are many studies on the vibrational characteristics of microplates employing linear elastic models or nonlinear elastic models with linear damping mechanisms. Furthermore, most of these studies are concerned with perfectly flat microplates without any geometric imperfections. For instance, in a study using a linear elastic model, Ashoori Movassagh and Mahmoodi (2013) studied the linear deflection of a microplate based on Kirchhoff plate theory and the modified strain-gradient elasticity theory. Hashemi and Samaei (2011) studied the linear buckling characteristics of micro/nonoplates as a result of in-plane forces through use of the nonlocal Mindlin plate theory. Further investigations was conducted by Wang, Zhou, Zhao, and Chen (2011), who analysed the linear micro-scale-dependent characteristics of Kirchhoff plates on the basis of the strain gradient theory. A linear vibration analysis was conducted by Jomehzadeh, Noori, and Saidi (2011), who examined the variations of natural frequencies of rectangular microplates with different system parameters employing the MCST. The investigations were continued by Nabian, Rezazadeh, Almassi, and Borgheei (2013), who examined the stability characteristics of doubly clamped functionally graded micro-scale plate subject to hydrostatic and electrostatic pressures. Evaluation of thermal effects on the free vibration of FG quadrilateral microplates. Ghorbani Shenas and Malekzadeh (2016) studied the linear vibration characteristics of functionally graded microplates in a thermal environment; they derived the eigenfrequency equations employing the modified strain gradient theory and analysed the thermal effects on the free vibrational behaviour of the microplate. Apart from the studies dealing with linear models, there are some studies of microplates in which a geometrically nonlinear model has been used. For example, Wang, Lin, and Zhou (2014) developed a size-dependent Kirchhoff plate model for the nonlinear bending investigation of circular microplates under distributed transverse forces employing MCST; they analysed the nonlinear static deflection of the microplate employing the Newton–Raphson iteration method. Ashoori and Mahmoodi (2017) developed a geometrically nonlinear first-order shear deformable model of a microplate employing the modified strain gradient theory as well as the von Kármán nonlinear strains.

The present study, for the first time, studies the *forced nonlinear* vibrational characteristics of a *geometrically imperfect viscoelastic shear deformable* microplate. More specifically, an accurate nonlinear micro-scale dependent model of the microplate is developed while accounting for: initial fabrication imperfection, geometric nonlinearities via von Kármán strain nonlinearities, damping nonlinearities via Kelvin–Voigt model, micro-scale size effects via MCST, and shear deformation via employing the third-order shear deformation theory. The developed complex nonlinear model is discretised via application of a two-dimensional Galerkin technique through use of hyperbolic and sinusoidal spatial trial functions for increased accuracy. A thorough numerical investigation is carried out via an efficient numerical algorithm based on a continuation technique. The nonlinear vibrational characteristics of the initially imperfect microsystem in the primary resonance region are studied and the effect of major contribution parameters such initial imperfection, micro length-scale parameter, and material viscosity are examined.

2. Equations of motion

Taking into account the fabrication imperfections, the nonlinear third-order shear deformation plate theory is employed in conjunction with the modified couple stress theory as well as the Kelvin–Voigt viscoelastic model (Ghayesh, 2018a; Ghayesh & Farokhi, 2018) to develop nonlinear model for a microplate consisting of geometric and damping nonlinear terms. To this end, a microplate of dimensions a and b, as shown in Fig. 1, is considered. The geometric parameters are defined within a Cartesian coordinate system (x, y, z), with dimension a being in the x direction and dimension b in the y direction. The microplate thickness is represented by h (in the z direction). Based on the third-order shear deformation theory, the microplate motion is described through use of five independent variables which are functions of x, y, and t, with t being time. These variables consist of three mid-plane displacements, i.e. w in the z direction, u in the x direction, and v in the y direction, as well as two mid-plane rotations, i.e. ϕ_1 and ϕ_2. In the present study, an initial geometric imperfection in the form of $w_0(x, y)$ is assumed in the z direction. In order to be able to examine the nonlinear resonant characteristics of the microsystem, a distributed harmonic excitation force, of amplitude F_1 and frequency Ω, is exerted on the microplate in the z direction. In the following, a geometrically imperfect viscoelastic nonlinear model of the microplate accounting for small-scale effects is developed.