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# Cascaded lattice Boltzmann method based on central moments for axisymmetric thermal flows including swirling effects

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ABSTRACT

A cascaded lattice Boltzmann (LB) approach based on central moments and multiple relaxation times to simulate thermal convective flows, which are driven by buoyancy forces and/or swirling effects, in the cylindrical coordinate system with axial symmetry is presented. In this regard, the dynamics of the axial and radial momentum components along with the pressure are represented by means of the 2D Navier-Stokes equations with geometric mass and momentum source terms in the pseudo Cartesian form, while the evolutions of the azimuthal momentum and the temperature field are each modeled by an advection-diffusion type equation with appropriate local source terms. Based on these, cascaded LB schemes involving three distribution functions are formulated to solve for the fluid motion in the meridian plane using a D2Q9 lattice, and to solve for the azimuthal momentum and the temperature field each using a D2Q5 lattice. The geometric mass and momentum source terms for the flow fields and the energy source term for the temperature field are included using a new symmetric operator splitting technique, via pre-collision and post-collision source steps around the cascaded collision step for each distribution function. These result in a particularly simple and compact formulation to directly represent the effect of various geometric source terms consistently in terms of changes in the appropriate zeroth and first order moments. Simulations of several complex buoyancy-driven thermal flows and including rotational effects in cylindrical geometries using the new axisymmetric cascaded LB schemes show good agreement with prior benchmark results for the structures of the velocity and thermal fields as well as the heat transfer rates given in terms of the Nusselt numbers. Furthermore, the method is shown to be second order accurate and significant improvements in numerical stability with the use of the cascaded LB formulation when compared to other collision models for axisymmetric flow simulations are demonstrated.



## 1. Introduction

Fluid motion in cylindrical coordinates with axial symmetry that is driven by rotational effects and/or thermal buoyancy effects arise widely in a number of engineering applications and geophysical contexts (e.g., [1–5]). Some examples of technological applications encountering heat and mass transfer effects in axisymmetric flows include pipeline systems, heat exchangers, solar energy conversion devices, crystal growth and material processing systems, electronic cooling equipment and turbomachinery. Computational methods play an important role for both fundamental studies of the fluid mechanics and heat transfer aspects and as predictive tools for engineering design of such systems. In general, fluid motion in cylindrical coordinates due to swirling effects and buoyancy forces, and accompanied by thermal and mass transport is three-dimensional (3D) in nature. Computational effort for such problems can be significantly reduced if axial symmetry, which arise in various contexts, can be exploited; in such cases the system of equations can be reduced to a set of quasi-two-dimensional (2D) problems in the meridian plane, where the simulations can be performed for broader ranges of the parameter spaces more efficiently. Traditionally, numerical schemes based on finite difference, finite volume or finite elements were constructed to solve the axisymmetric Navier-Stokes (NS) equations for the fluid flow along with the advection-diffusion equation for the energy transport (e.g., [6,7]).

On the other hand, lattice Boltzmann (LB) methods, which arise as minimal kinetic models of the Boltzmann equation, has attracted much attention and application to a wide range of fluid flows and heat and mass transfer problems [8–12]. They can be characterized as mesoscopic computational approaches, which

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have the following unique features and advantages: Its streaming step is linear and exact and all nonlinearity is modeled locally in the collision step; by contrast, the convective term in the NS equation is nonlinear and nonlocal. As a result, the pressure field is obtained locally in the LB methods, circumventing the need for the solution of the time consuming elliptic Poisson equation as in traditional methods. The exact-advection in the streaming step combined with the collision step based on a relaxation model leads to a second order accurate method with relatively low numerical dissipation. The kinetic model for the collision step can be tailored to introduce additional physics as necessary and its additional degree of freedom can be tuned to improve numerical stability. Various boundary conditions for complex geometries can be represented using relatively simple rules for the particle populations. Finally, the locality of the method makes it amenable for almost ideal implementation on parallel computers for large scale flow simulations.

Following an approach for the solution of the Boltzmann equation in cylindrical coordinates [13], during the last two decades, various LB schemes for athermal flows (i.e., without heat transfer effects) have been introduced [14–22]. These approaches can be categorized according to the following: (i) Coordinate transformation method [14–18], in which the axisymmetric mass and momentum equations are reformulated as quasi-2D flow equations in the Cartesian forms with additional geometric source terms and then solved using a LB scheme. (ii) Vorticity-stream function approach [19], where LB models are introduced to simulate flows in the cylindrical coordinates written in terms of the vorticity and stream function equations. (iii) Radius-weighted formulation [20], in which a simplified LB method is derived from a discretization of the continuous Boltzmann equation in cylindrical coordinates recast in a radius-weighted form. An analysis of these axisymmetric LB models were performed by [21]. Generally, these approaches use a popular single relaxation time (SRT) model for the representation of the collision step in the LB scheme.

Further progress in the LB methods for the simulation of axisymmetric thermal flows have been reported in various studies [23–32]. Earlier LB models in this regard [23,24] used an hybrid approach, in which the energy equation was solved via a finite difference scheme. Later, [25,26] solved the axisymmetric equation for the temperature field written in terms of a pseudo-2D advection-diffusion equation with a source term using a LB scheme based on a separation distribution function from that for the flow field. On the other hand, [27,28] extended the radius-weighted formulation approach for axisymmetric fluid flow [20] for the simulation of thermal energy transport. A fractional-step based LB flux solver for axisymmetric thermal flow was presented in [30]. All these approaches were based on the common SRT model [33], in which, during the collision step, the distribution functions relax to their local equilibria using a single relaxation parameter. This was further extended by the introduction of two tunable parameters as coefficients to the additional gradient terms in the equilibrium distribution functions [32]. Generally, SRT based LB schemes are known to be susceptible to numerical instabilities for convection-dominated flows or fluids with relatively low values of transport coefficients. In order to address this issue, the collision step based on a multiple relaxation time (MRT) model [34] has been constructed, in which raw moments of different orders relax at different rates. Few MRT LB schemes for axisymmetric thermal convective flows have recently been developed [26,29,30].

On the other hand, further improvements to the collision step enhancing the flow and thermal transport modeling capabilities can be achieved via the introduction of the cascaded collision model [35]. In this approach, the effects of collisions are represented in terms of relaxation of different orders of central moments, which are obtained by shifting the particle velocity, by the local fluid velocity at different rates. As the collision model is prescribed based on a local moving frame of reference, the relaxation steps for successive higher order moments exhibit a cascaded structure. The cascaded collision formulation was shown to be equivalent to considering relaxation to a generalized equilibrium in the rest or lattice frame of reference [36], and was augmented with forcing terms in 2D and 3D in [37,38]. Improvements in the numerical properties achieved using such advanced cascaded collision models based on central moments were recently demonstrated [39,40]. A modified formulation based on central moments involving relaxation to discrete equilibria rather than continuous Maxwellian equilibria was also proposed [41]. In order to accelerate convergence of steady flows, a preconditioned cascaded LB method was constructed and studied in [42], and whose Galilean invariance properties were significantly improved via corrections to equilibria in [43]. The cascaded LB scheme has recently been extended for simulating flows with heat transfer in 2D [44,45] and in 3D in our recent work [46]. Some related recent papers that discuss about the forcing schemes as well as thermal sources are [47–50]. However, for axisymmetric thermal convective flows including rotational effects, no such advanced LB schemes are available in the literature.

In this work, we present a new cascaded LB formulation for thermal flows in cylindrical coordinates with axial symmetry, and including rotational effects. The mass, momentum (i.e., for the axial, radial and azimuthal components) and energy equations rewritten in pseudo-2D Cartesian forms in the meridian plane contain additional geometric source terms, which are included in the respective cascaded LB schemes via a novel symmetric time-split formulation that we developed recently [50]. In this approach, three separate distribution functions are considered: one for the density, axial and radial momentum components, the second for the azimuthal momentum component and, finally, the third for the temperature field. Each of the three distribution functions evolves according to a cascaded LB scheme. For this triple distribution functions framework, a two-dimensional, nine velocity (D2Q9) model is used to solve for the axisymmetric NS equations for the axial and radial momentum components, while a two-dimensional five velocity (D2Q5) model is employed to compute the azimuthal momentum and the temperature field, both of whose evolution are represented by advection-diffusion equations with source terms. The use of symmetric operator split formulations based on pre-collision and post-collision source steps with a half time step in each case for incorporating the geometric source terms for axisymmetric thermal flows including swirl effects leads to a particulary simplified formulation. Such an approach is consistent with the classical Strang splitting. As will be shown later in this paper, the application of central moments based cascaded LB schemes using MRT can significantly enhance the numerical stability of axisymmetric LB simulations allowing broader range of parameter spaces more efficiently, and the use of symmetric operator splitting yields a scheme that is second order accurate [50]. Such an axisymmetric cascaded LB formulation for the simulation of thermally stratified and/or rotating flows in cylindrical geometries can lead to reduced computational and memory costs when compared to a 3D cascaded LB scheme. Several numerical axisymmetric benchmark problems focusing on buoyancy-driven flows and rotational effects are considered to validate our operator-split axisymmetric cascaded LB schemes for thermal flows. These include the Taylor-Couette flow, natural convection in an annulus between two co-axial vertical cylinders, Rayleigh-Benard convection in a vertical cylinder, cylindrical lid-driven cavity flow, mixed convection in a tall vertical annulus and melt flow during Czochralski crystal growth in a vertical rotating cylinder.

This paper is organized as follows. In the next section (Section 2), cascaded LB methods for axisymmetric thermal flows