International Journal of Heat and Mass Transfer 128 (2019) 1026–1032

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Mathematical modelling of frequency and force impacts on averaged metal flows in alternating magnetic field

I.L. Nikulin *, A.V. Perminov

Perm National Research Polytechnic University, 614990 Perm, Russia

A R T I C L E I N F O

Article history:
Received 20 February 2018
Received in revised form 15 July 2018
Accepted 29 August 2018

Keywords:
Induction melting
High frequency magnetic field
Eddy currents
Lorentz force
Forced convection
Averaged flow

A B S T R A C T

The averaged quasi-steady flow patterns of metal melt situated in alternating magnetic field are studied numerically. It is taken into account the buoyancy due to non-uniform heating, the intensive radiation heatsink from the melt surface and the Lorentz forces arising in conductor in the alternating magnetic field. The governing equations of the suggested mathematical model are given and the model validations are described. The flow regime maps are plotted based on analysis of melt velocity dependence on the Hartmann number and the parameter of magnetic field diffusion. Flow patterns corresponding each parameter area at the map are given and interpreted. The way of acceleration of melt motion computation is suggested.

1. Introduction

Induction melting is an effective technique for the melting of nickel heat resistant alloys providing high uniformity of the alloy temperature and of the distribution of alloying species. This uniformity is achieved by steering the melt during melting by electromagnetic forces induced in the melt by the interaction of eddy currents and magnetic fields. The induction melting process at the "Proton – Perm Motors" (Perm, Russia) takes places in the vacuum casting furnace; its schematic is shown in Fig. 1. Ceramic crucible filled with the melt is installed in the inductor (a helix water-cooled copper conductor) connected to a current source. The alternating current in the inductor generates an alternating magnetic field which excites eddy currents in the molten metal. The melt is heated by Joule heat and melted. The temperature of the melt is measured by a pyrometer during melting. The shutter protects the pyrometer viewport of the vacuum chamber from the metal spatters appearing at the beginning of melting. The control system samples the temperature data from the pyrometer, drives the shutter and controls the current intensity in the inductor.

One of the problems of this technological process is the formation of a thin oxide layer (scab) on the surface of the heat resistant alloy. Firstly, the scab appearing during casting leads to defects and worsens the alloy production. Secondly, the oxide scab has the emissivity greater than that of the molten metal, it distorts the alloy temperature readouts from the pyrometer.

Possible solution of the given problem is the organization of the fluid motion such that the melt flows at the surface are directed from the center to the walls. In this way, the scab will be driven to the crucible walls and glued to it (the scab gluing to the crucible walls is an experimental fact) and the melt surface will be cleaned.

The possibility of fluid motion organization is based on the understanding of mechanisms driving the metal flows in alternating magnetic fields (AMF). The laboratory investigation of the metal melt flows induced in AMF is carried out for a long time, the information about the flow structure may be generalized as follows.

The alternating magnetic field is penetrating into conductive material for a finite depth δ depending on material properties and AMF frequency [1–3].

$$\delta = \left(\frac{1}{2}\mu_a\sigma\Omega\right)^{-\frac{1}{2}}, \qquad (1)$$

where μ_a is the permeability, σ is the electrical conductivity, Ω is the inductor current cyclic frequency, the joule heat sources and body Lorentz forces, which generated by eddy currents and interaction of eddy currents with magnetic field respectively, are localized in this depth.

The eddy currents magnetic field structure is determined by the dimensionless parameter of magnetic field diffusion (or shield

---

* Corresponding author.
E-mail address: nil@pstu.ru (I.L. Nikulin).

https://doi.org/10.1016/j.ijheatmasstransfer.2018.08.130
0017-9310/© 2018 Published by Elsevier Ltd.

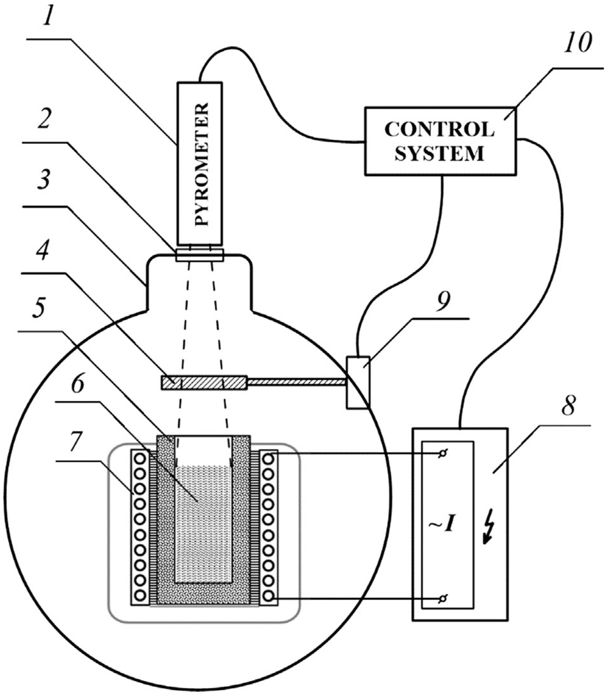

Fig. 1. Schematic presentation of the vacuum casting furnace: *1* – pyrometer, *2* – viewport, *3* – vacuum chamber, *4* – shutter protecting the viewport from melt spatters, *5* – crucible, *6* – melt, *7* – inductor, *8* – power supply, *9* – shutter driving device, *10* – control system. The optical path of the pyrometer is dashed-line marked.

parameter, or dimensionless frequency, or magnetic Reynolds number) [4–6],

$$D_H = \mu_a \sigma R^2 \Omega, \qquad (2)$$

where R is the internal radius of the metal filled crucible.

The general means of control of induction processes of heating and melting (if it takes place) are the choice of inductor geometry [7–11], and the selection of AMF frequency [9,12–15]. There is a possibility to make some special mode of coil switching [16,17]. For high enough frequencies of the AMF ($D_H > 1$) it is possible to neglect the motion of conductive melt to internal magnetic field distribution [18].

During the induction melting the type of melt flow in the crucible is turbulent [4–6,19], but there are main averaged (quasi-stationary or statistically stationary) macroscopic flows with some distributions (quite significant in the natural experiment); for frequencies lesser some value the double torous flow structure is observed [4–6,19–21]. The velocity linearly depends on AMF induction amplitude (or current intensity in the inductor coil), this dependence was observed both for fusible alloys (mercury [4], GaInSn [5], Wood's alloy [22]) and metals with high enough melting temperature such as TiAl alloy [1] and steel [21].

Many of the authors use isothermal approach to model melt convection. It is reasonable for fusible alloys or for experiments, which are carried out in insulated devices, where there is no surface heatsink and buoyancy does not play significant role; in such systems fluid motion is generated by Lorentz force only and it is very intensive. In the greatest number of works the results of modelling of AMF-generated flows are practical and concerned to the specific device mode; the generalizations of flow pattern, and diagrams of steady regimes as in [23] are rare. But the information about the flows stability may be useful for design of the induction devices or its technological regimes.

The most important dimensionless parameters [1] controlling fluid flow in electromagnetic field are the parameter of magnetic field diffusion defined in Eq. (1) and the Hartmann number

$$Ha = \sqrt{\frac{\mu_a H_0^2 R^2}{\rho \nu^2}}, \qquad (3)$$

where H_0 is characteristic magnetic field, ρ is the density, ν is kinematic viscosity.

The purposes of this work are

- to define ranges of dimensionless parameters (parameter of magnetic field diffusion, which is determined by the frequency, and the Hartmann number characterizing magnetic field strength), where averaged quasi-steady flows exist in system with intensive radiational heatsink from the melt surface;
- to reveal the impact of thermophysical melt properties on flow patterns.

2. Mathematical model

The mathematical model has following assumptions

- the system has axial symmetry (this approach is used widely, see [5,12,24]);
- melt surface is flat;
- melt convection does not affect magnetic fields and eddy currents (corresponding estimations were made in work [25]), and it is possible to solve the problem of diffusion magnetic field in the conductive melt and the heat and mass transfer problem separately;

The eddy currents magnetic field represents sum of harmonic amplitudes H_1 and H_2

$$\boldsymbol{H} = \boldsymbol{H}_1 \sin\Omega t + \boldsymbol{H}_2 \cos\Omega t. \qquad (4)$$

The governing equation system for these amplitudes with boundary conditions is

$$\begin{aligned} \nabla^2 \boldsymbol{H}_1 &= D_H(\boldsymbol{H}_2 - \boldsymbol{H}^{out}), \\ \nabla^2 \boldsymbol{H}_2 &= -D_H \boldsymbol{H}_1, \end{aligned} \qquad (5)$$

$$H_r|_{r=0} = \left.\frac{\partial H_z}{\partial r}\right|_{r=0} = 0, \quad \boldsymbol{H}_{1,2}|_{z,r\to\infty} = 0, \qquad (6)$$

where $\boldsymbol{H}^{out}$ is the amplitude of the inductor magnetic field, H_r and H_z are radial and axial component of $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ respectively.

The solution of system (5) with boundary (6) and AMF diffusion parameter (2) lets to compute the averaged curl of the electrodynamic Lorentz [8]

$$F = \frac{1}{2}\nabla \times [(\boldsymbol{H}_1\nabla)\boldsymbol{H}_1 + (\boldsymbol{H}_2\nabla)\boldsymbol{H}_2] \qquad (7)$$

and the heat power generated by eddy currents

$$q = \frac{1}{2}[(\nabla \times \boldsymbol{H}_1)^2 + (\nabla \times \boldsymbol{H}_2)^2] \qquad (8)$$

The method of solution of Eqs. (2)–(6) was considered in details in work [8].

The dimensionless equations for the momentum transfer are written in the terms of the stream function and the vorticity with taking into account equation of continuity