International Journal of Heat and Mass Transfer 128 (2019) 1149–1156

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A fractal model of effective thermal conductivity for porous media with various liquid saturation

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ARTICLE INFO

Article history:
Received 2 June 2018
Received in revised form 1 September 2018
Accepted 15 September 2018

Keywords:
Effective thermal conductivity
Porous media
Fractal
Saturation

ABSTRACT

Thermal conduction in porous media has received wide attention in science and engineering in the past decades. Previous models of the effective thermal conductivity of porous media contain empirical parameters typically with ambiguous or even no physical rationales. This study proposes a theoretical model of effective thermal conductivity in porous media with various liquid saturation based on the fractal geometry theory. This theoretical model considers geometrical parameters of porous media, including porosity, liquid saturation, fractal dimensions for both the granular matrix and liquid phases, and tortuosity fractal dimension of the liquid phase. Effects of these geometrical parameters on the effective thermal conductivity of porous media are also evaluated. This proposed fractal model has been validated using published experimental data, compared with previous models, and thus provides a physics-based theoretical model that can provide insight to geoscience and thermophysics studies on thermal conduction in porous media.

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1. Introduction

Thermal conduction in porous media is essential to many applications, such as exploiting and utilizing geothermal energy [1,2], determining heat flow in hydrothermal systems [3], reconstructing past climate [4], modeling hydrocarbon formation processes [5], and investigating potential nuclear wastes [6]. As thermal conductivity is a key thermophysical parameter in characterizing the heat transfer process of porous media, accurate evaluation of thermal conductivity is important in many engineering and science fields [7–10].

Existing effective thermal conductivity models are mostly classical mixing laws. The well-known weighted arithmetic and harmonic means are used as the upper and lower limits of thermal conductivity for two-phase saturated porous media (i.e., the parallel and series models) [1,7]. The Maxwell-Eucken model is often used to characterize the thermal conductivity of porous media, where spherical pores are assumed to be widely dispersed in a continuous medium. The effective medium theory provides approximated thermal conductivity for macroscopically homogeneous and isotropic media containing randomly distributed grains and pores [11], which can be predicted based on the thermal-electrical analogy theory [12].

Recently, Carson et al. [13] classified the materials into two types (i.e., internal porosity, external porosity) and proposed a thermal conductivity range for isotropic porous media. Wang et al. [14] derived a unified equation for five fundamental effective thermal conductivity structural models (i.e., series, parallel, two forms of Maxwell-Eucken, and effective medium theory), although the determination of appropriate parameters in these models remains a challenge. Gong et al. [15] proposed another analytical expression unifying above five fundamental effective thermal conductivity structural models without any weighting parameter. However, none of these above-mentioned theoretical models considers the impacts of porous media's microstructure, such as pore geometry, pore size distribution, and the tortuosity of micropores on thermal conduction. Additionally, many numerical approaches, such as Monte Carlo method [16], finite element method [17], and lattice Boltzmann method [18], are developed to study the thermal conductivity of porous media. These results are often expressed as semi-empirical functions.

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https://doi.org/10.1016/j.ijheatmasstransfer.2018.09.072
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The fractal geometry theory has been widely used to study the transport properties of disordered porous media, such as thermal conductivity [19,20], spontaneous capillary imbibition [21–23], permeability [24–27], and electrical conductivity [28,29]. Yu and Cheng [30] developed a model to calculate the effective thermal conductivity of bi-dispersed porous media based on the fractal theory and electrical analogy technique. Ma et al. [31] proposed a fractal method to predict the effective thermal conductivity of saturated and unsaturated porous media based on the Sierpinski carpet model. The Sierpinski carpet fractal model was then generalized by Feng et al. [32,33]. Jin et al. [12] applied the Sierpinski carpet fractal model proposed by Feng et al. [32,33] to calculate the effective thermal conductivity of autoclaved aerated concrete with different moisture content. Kou et al. [34] presented a fractal analysis on the effective thermal conductivity for saturated and unsaturated porous media without any empirical constant. Pia et al. [35,36] proposed an intermingled fractal model for the thermal conductivity of porous media considering microstructures. Miao et al. [37] proposed an analytical expression of effective thermal conductivity for saturated dual-porosity media based on fractal characteristics of pores and fractures. Furthermore, the fractal models for heat conduction of porous media have been proposed to study the thermal conductivity of nanofluids, by considering the fractal characteristics of nanoparticle sizes and the heat convection through the Brownian motion of nanoparticles in fluids [38]. Wei et al. [39] proposed a theoretical effective thermal conductivity model for nanofluids based on the fractal distribution characteristics of nanoparticle aggregation.

The thermal conductivity of porous media is strongly influenced by the media's microstructural features, which can be well characterized by the fractal geometry. However, most previous fractal models of effective thermal conductivity are proposed by the thermal-electrical analogy technique. In this study, a theoretical fractal model of effective thermal conductivity for both saturated and unsaturated porous media is presented originated from the Laplace's Equation. The proposed fractal model is validated by published experimental data, and the effects of various geometrical parameters on the effective thermal conductivity of porous media are also discussed.

2. A fractal thermal conductivity model for saturated porous media

The local thermal flux $\boldsymbol{q}$ in a porous medium with a temperature gradient ∇T is given by [40]

$$\boldsymbol{q} = -k\nabla T, \tag{1}$$

where k is the thermal conductivity and $k = k_f$ for the pore fluid phase and $k = k_s$ for the solid phase. Eq. (1) is defined by the classical Fourier's law. The ensemble volume-averaged solution of Eq. (1) is given by

$$\bar{\boldsymbol{q}} = -k_e\overline{\nabla T}, \tag{2}$$

where k_e is the effective thermal conductivity of the porous medium, $\bar{\boldsymbol{q}}$ is the ensemble volume average of local thermal flux, and $\overline{\nabla T}$ is the ensemble volume average of the local temperature gradient. These ensemble volume-averaged quantities are given by [11]

$$\bar{\boldsymbol{q}} = \frac{1}{V}\int_V \boldsymbol{q}dV, \tag{3}$$

$$\overline{\nabla T} = \frac{1}{V}\int_V \nabla T dV, \tag{4}$$

where V is the total volume of the porous medium.

Then, the ensemble volume average of local thermal flux can be rewritten as [40,41]

$$\begin{aligned}\bar{\boldsymbol{q}} &= \tfrac{1}{V}\int_{V_p}\boldsymbol{q}dV + \tfrac{1}{V}\textstyle\sum_k\int_{V_k}\boldsymbol{q}dV\\ &= -\tfrac{k_f}{V}\int_{V_p}\nabla T dV - \tfrac{k_s}{V}\textstyle\sum_k\int_{V_k}\nabla T dV\,,\\ &= -k_f\overline{\nabla T} - \tfrac{k_s-k_f}{V}\textstyle\sum_k\int_{V_k}\nabla T dV\end{aligned} \tag{5}$$

where V_p is the pore volume and V_k is the volume of a grain. It is assumed that the grains have identical thermal conductivity k_s. Thus, it becomes a classical problem of temperature distribution in a single sphere (grain) with a diameter λ and thermal conductivity k_s immersed in a continuous medium (pore phase) with thermal conductivity k_f. This system is subjected to a steady-state temperature gradient $\overline{\nabla T}$ in the direction of the z-axis, as shown in Fig. 1.

In the steady-state condition, the temperature distribution of a single particle within a uniform medium obeys the Laplace's Equation in a spherical coordinate system as [13]:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\varphi^2} = 0, \tag{6}$$

where θ is the polar angle, φ is the azimuthal angle, and r is the distance (radius) from a point to the origin. Assuming symmetry about the z-axis such that the temperature distribution is independent of φ, Eq. (6) can be rewritten as [15]:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) = 0, \tag{7}$$

a general solution of Eq. (7) is [13,15]

$$T = A + \frac{B}{r} + Cr\cos\theta + \frac{D}{r^2}\cos\theta, \tag{8}$$

where A, B, C, D can be obtained by the following boundary conditions:

(1) at $r = 0$, $T_s \neq \infty$;
(2) at $r = \frac{\lambda}{2}$, $k_s\frac{\partial T_s}{\partial r} = k_f\frac{\partial T_f}{\partial r}$ and $\frac{\partial T_s}{\partial\theta} = \frac{\partial T_f}{\partial\theta}$;
(3) at $r \gg \frac{\lambda}{2}$, $T_f = \overline{\nabla T}\cdot\boldsymbol{z} = \overline{\nabla T}r\cos\theta$.

With boundary conditions, the temperature distribution within the sphere can be gotten by solving Eq. (8):

$$T_s = \frac{3k_f}{k_s + 2k_f}\overline{\nabla T}r\cos\theta. \tag{9}$$

And the temperature distribution outside the sphere is:

$$T_f = r\cos\theta - \overline{\nabla T}\frac{\lambda^3}{8}\frac{k_s - k_f}{k_s + 2k_f}\frac{\cos\theta}{r^2}. \tag{10}$$

Based on Eq. (9), the thermal flux through a single grain is given by

$$\boldsymbol{q_k} = -(k_s - k_f)\int_{V_k}\nabla T dV = -\frac{\pi\lambda^3}{6}\left[\frac{3k_f(k_s - k_f)}{k_s + 2k_f}\right]\overline{\nabla T}. \tag{11}$$

Additionally, it has been shown that the irregular nature of natural porous media follows the fractal scaling law. Thus, the grain size distribution is assumed to obey the following relationship with grains diameters from λ to $\lambda + d\lambda$ [42]:

$$-dN = D_{fs}\lambda_{\max}^{D_{fs}}\lambda^{-(D_{fs}+1)}d\lambda, \tag{12}$$

where $\lambda_{\max}$ is the maximum grain diameter, D_{fs} is the fractal dimension of grains, and $dN < 0$ suggests that the number of grains decreases with increasing of the grain diameter. For natural porous media, the relationship between the fractal dimension D_{fs} and porosity ϕ can be expressed as [42]

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