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Some features of solving an inverse problem on identification of material properties of functionally graded pyroelectrics

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ABSTRACT

In the paper, we propose an approach to solving an inverse problem of identifying material characteristics of a functionally graded thermo-piezoelectric body. The operator reciprocity equations of the first kind are obtained in order to solve the problem formulated on the basis of the iterative process. As an example, the inverse problem of thermo-electroelasticity for a pyroelectric rod is investigated. We have carried out the computational experiments on restoration of the rod characteristics with various laws of inhomogeneity including those modeling layered and functionally graded coatings.

1. Introduction

Currently, pyroelectric materials are widely used in the construction of various diagnostic instruments and temperature sensors due to the presence of the effect of mutual coupling of thermal, electric and elastic fields [1–4]. To describe the mechanical behavior of pyromaterials, the equations of thermo-electroelasticity are often used, which were first obtained by Mindlin [5,6] in the early 1960s. Note that the problems of thermo-electroelasticity for homogeneous and piecewise homogeneous bodies have been studied in sufficient detail, and their analytical solutions have been obtained [7–10]. The model of inhomogeneous electroelasticity was first based on the model of a layered structure [11].

Functionally graded pyromaterials (FGPM) are widely used in various devices based on the pyrotechnic technologies in order to get the required properties. FGPM are composites with variable physical properties, which avoid jumps of material characteristics across the interface, inherent in laminated materials. In this case, the material characteristics of FGPM are not constants, but some functions of spatial coordinates. Investigations of thermo-electroelasticity problems for graded materials have been carried out mainly for power and exponential inhomogeneity laws [12–16]. In [17], an approach to the solution of problems of thermo-electroelasticity was proposed for arbitrary distribution laws of inhomogeneity. The problem of thermal shock in a functionally graded thermo-piezoelectric layer was solved by leading to the system of the Fredholm integral equations of the second kind written for the Laplace transforms and finding the actual space on the basis of the theory of residues.

To describe the mechanical behavior of devices containing inhomogeneous pyromaterials, it is necessary to know the material characteristics representing certain functions of the coordinates. In this case, measurements of material characteristics based on macroexperiments are impossible. A task of identifying laws of variation of inhomogeneous characteristics of pyromaterials refers to a coefficient inverse problem of thermo-electroelasticity [18].

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Nomenclature

S	body surface	p	Laplace transform parameter
D_i	components of the electric induction vector	g_i	components of the displacement vector measured at the boundary S_σ
u_i	components of the displacement vector	f	temperature measured at the boundary S_q
φ	electrical potential	$\tilde{v}_i, \tilde{\vartheta}, \tilde{\psi}$	smooth test functions
θ	temperature increment	l	rod length
c_{ijkl}	components of the elastic modulus tensor	x_3	axis along which the rod is polarized
ρ	density	z	dimensionless coordinate
k_{ij}	components of the heat conductivity tensor	U	dimensionless rod displacement
A_ε	specific volume heat capacity	W	dimensionless rod temperature
γ_{ij}	thermal stress tensor components	V	dimensionless electric potential of the rod
e_{kij}	components of the tensor of piezoelectric modules	τ	dimensionless time
M_{ij}	components of the permittivity tensor	$\delta_1, \delta_2, \delta_3$	dimensionless coupling parameters
g_i	components of the pyroelectric coefficient tensor	ε	the ratio of the characteristic times of sound and thermal oscillations
t	time	β	limit value of the residual functional
q	heat flux density	α	amount of noise
p_i	components of the vector of mechanical load applied to the body	ω	random value
$H(\tau)$	Heaviside function		
n_j	components of the unit vector of the outer normal to S		

In practice, two types of inverse problems statement in mathematical physics are widespread. For the first type, additional data is assumed to be known at internal points of a body at some moment of time; for the second type, it is assumed to be given on boundary part only, at some time interval. The second type of inverse problem statement is the most realizable experimentally.

Coefficient inverse problems of thermo-electroelasticity of the second type are ill-posed and non-linear problems. Therefore, development of stable algorithms of solving such problems is actual problem. Currently, there is a lack of works on inverse problems of the mechanics of coupled field; let us mention some of them concerning electroelasticity [19–21], thermoelasticity [22,23].

The most common approach to solving inverse problems of mathematical physics is based on their reduction to residual functional minimization; to do this, as a rule, they use gradient methods [24,25], or genetic algorithms [14]. However, one of the drawbacks of such methods is that the with an increase in the number of unknowns delivering a minimum of the objective function, the amount of computations grows.

In a number of works, an approach based on the construction of an iterative process [26] has been applied to solve inverse problems of the mechanics of coupled fields. At each stage of the iterative process one should solve the linearized operator equations of the first kind. Using this approach, a number of problems on identification of inhomogeneous characteristics in linear electroelastic [19–21], thermoelastic [27–30], and elastic prestressed bodies [31–35] was successfully carried out. In [20], for solving the electroelasticity inverse problem, a weak formulation is formulated, and the problem of reconstructing the law of variation of the compliance modulus of the rod within steady-state longitudinal oscillations is solved. In [21], a method for reconstructing the properties of a non-uniformly thick electroelastic layer from the analysis of the plane problem on steady oscillations is proposed. With the help of the Fourier transform, two simpler (unbound) boundary value problems with respect to the averaged displacement and potential are solved. The solution of the inverse problem is constructed on the basis of the linearization method and the iterative process with solving the system of the Fredholm integral equations of the first kind. This work contains examples of simultaneous reconstruction of the inhomogeneity laws of the piezoelectric module and the elastic modulus. In [27,28], the solution of the inverse thermoelasticity problem for a rod is presented. Two loading types are used: mechanical and thermal. As additional data, temperature or displacement measured on a part of the body boundary were considered.

When solving inverse problems, it is important to consider conditions that ensure the uniqueness of simultaneous identification of two or more material characteristics. In [36,37], some types of statements of inverse heat conduction problems on simultaneous identification of the thermal conductivity and specific heat capacity are considered and investigated. Such statements make it possible to solve the inverse problem of heat conduction in the framework of one experiment. Note that it is difficult to realize the data measurement in the way, as it was represented in the above-mentioned works. In [30], the authors proposed another approach. The inverse problem of simultaneous identification of the thermal conductivity coefficient and specific heat capacity is solved on the basis of the additional information obtained in the course of two experiments. Similarly, the inverse problem of electroelasticity is solved in [21].

In the present paper, to solve the coefficient inverse problems of thermo-electroelasticity, we generalize our previously developed approaches to solving inverse problems of thermoelasticity and electroelasticity. For this, we have derived the weak statement of the thermoelectroelasticity problem in the Laplace transforms. On the basis of the weak statement in the Laplace transforms and the linearization method, the operator equations of the first kind were obtained.

As an example of application of the approach proposed, the inverse problem for a thermo-piezoelectric rod was considered. After applying the Laplace transform, the direct problem of thermo-electroelasticity for the rod were solved by reduction to a system of the Fredholm integral equations of the second kind, similarly to [20]; the solutions were constructed in the form of rational functions for the transforms, and the actual space was found on the basis of the theory of residues.

To treat the inverse problem of thermo-electroelasticity for a rod, the Fredholm integral equations of the first kind were solved at each stage of the iterative process. The computational experiments were carried out to restore the inhomogeneous characteristics of the rod, including those modeling layered and functionally graded coatings that have heat-protective properties [38]. In the first series of experiments, we restored only one of the rod's