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Inverse kinematics of a 5-axis hybrid robot with non-singular tool path generation

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ABSTRACT

This paper deals with non-singular tool path generation of a 5-axis hybrid robot named TriMule, which is designed for large part machining in situ. It is observed that at a singularity pose sudden changes occur in rotation of the C-axis and lengths of three telescopic legs. It is found that when the tool axis rotates about the axis normal to the plane expanded by the tool axis and the singular axis, the singular axis itself is forced to rotate simultaneously about the same axis in the opposite direction. This exploration enables the minimum rotation angle of the tool axis to be determined accurately for avoiding singularity and reducing machined surface errors caused by tool axis modification, leading to the development of an algorithm for non-singular tool path generation by modifying a partial set of the control points of B-splines. Both simulation and experiment on a prototype machine are carried out to verify the effectiveness of this approach.

1. Introduction

Parallel kinematic machines composed of a 1T2R (T—Translation, R—Rotation) parallel mechanism plus a A/C type wrist serially connected to the platform exhibit desirable performance in terms of rigidity, accuracy, work envelop and reconfigurability. They are therefore suitable to be built as robotized modules for large part manufacturing in situ, drilling, riveting and high-speed milling for example [1]. This statement can be exemplified by very successful applications of the well-known Tricept robots [2]. The similar modules with hybrid architecture are the Exechon [3], TriMule [4], George V [5] and its variant [6]. Similar to the conventional A/C type 5-axis machines with serial architectures, the above mentioned parallel kinematic machines (or hybrid robots) suffer from singularity problem which occurs when the tool axis is coincident with a special axis known as singular axis, leading to sudden changes in rotary and translational drivers, thus resulting in errors on machined surface, and even damage to machines themselves.

In the last two decades, intensive investigations have been carried out towards non-singular tool path generation of the conventional A/C type 5-axis machines where the singularity axis is fixed in space. Various concepts have been proposed for visually and quantitatively defining and detecting the singular domain, for instance, singular cone [7], singular ring [8,9], admissible orientation domain [10] and acceptable texture orientation region [11] in 2/3D representations. From a kinematic viewpoint, the methods in dealing with singularity problem can be roughly classified into two categories that can be performed in either the Cartesian space or the joint space. For the methods belonging to the first category, the refined subsequence of tool axis vectors falling into a specified singular cone are forced to rotate about a special axis while keeping full sequence of tool tip vectors unchanged. In realistic implementation, several CAD/CAM based algorithms have been developed to generate the refined sequence of tool axis vectors using single or double third order B-splines [8,9]. This allows the parameterized tool axis vectors to be modified by rotating the corresponding control point vectors. Here, the rotation can be represented by either the Roderigues formula or quaternion. The approach to determining the adequate tool orientations is also studied with the goal to minimize the machining error in the neighborhood of singularity [12]. The non-singular tool path can also be generated in post processing using the methods falling into the second category. In this regard, the refined sequence of C-axis commands is modified first by means of linear interpolation using the previous and current cutter locations in a recursive manner while keeping those associated with the other rotary axis (i.e. the A or B axis) and tool tip vectors unchanged. This allows in turn the tool axis vector, thus the commands of translational drives, to be modified accordingly *via* inverse kinematics [13,14]. In addition, some interesting work has been conducted for singularity avoidance by

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adjusting the workpiece pose with respect to that of the worktable [15,16].

In recent years, tremendous work has been carried out on the design and development of 5-axis hybrid robots having A/C type wrist, the Tricept, the Exechon and the TriMule for example. The relevant work primarily focuses on type synthesis [4,17,18], kinetostatic analysis [19–20], kinematic optimization [21–27], and CNC controller development [28–30], etc. Unfortunately, little attention has been paid to non-singular tool path planning though singularity problem can be avoided, to some extent, by placing such a robot a tilt angle with respect to the workpiece frame at the cost of reducing the useful workspace envelop [23,31]. It should be pointed out that unlike a conventional A/C type 5-axis machine tool where the singular axis is fixed in space, the singular axis of a 5-axis hybrid robot having A/C type wrist varies with system configurations, leading to the change of the singular axis when the tool axis is modified for singularity avoidance. This feature brings two important issues to be investigated in non-singular tool path generation: (1) how to determine the axis about which the tool axis needs to rotate an angle for singularity avoidance, and (2) how to determine the exact value of the angle that allows the machined surface errors caused by tool axis modification to be minimized.

Motivated by the practical needs arising from the abovementioned two issues and by taking the TriMule robot [4] as an example, this paper investigates non-singular tool path generation of the hybrid robots with AC type wrist. The remainder of the paper is organized as followers. Having reviewed the methods for non-singular tool path generation of the conventional A/C type 5-axis machine tools and addressed two issues to be investigated for the 5-axis hybrid robots having A/C type wrist in Section 1, inverse displacement analysis of the robot is carried out in Section 2 with the mission to investigate behaviours of the joint variables in the neighbourhood of singularity configurations. In Section 3, the relationship between rotations of the singular axis and tool axis is revealed, leading to the development of an algorithm for non-singular tool path generation by modifying a partial set of the control points of B-splines. In Section 4, both simulation and experiment on a prototype machine are carried out to verify the effectiveness of the proposed approach before conclusions are drawn in Section 5.

2. Inverse kinematics and singularity analysis

In this section, inverse displacement analysis will be carried out after a brief introduction to the structure of the TriMule robot. This is followed by the investigation into behaviours of the actuated joint variables in the neighbourhood of singular configurations.

2.1. System description

Fig. 1 shows a 3D view of the TriMule robot, which essentially consists of a 1T2R spatial parallel mechanism for positioning and a A/C type wrist attached to the platform *via* a trust bearing. The spatial parallel mechanism features a 6-DOF U$\underline{\text{P}}$S limb plus a 2-DOF planar parallel mechanism comprising two actuated R$\underline{\text{P}}$S limbs and a passive RP limb in between with its one extremity being rigidly fixed to the platform. The base link of the planar parallel mechanism is connected by a pair of R joints with the machine frame. Here, R, P, U, and S denote revolute, prismatic, universal, and spherical joints, and the underlined $\underline{\text{P}}$ denotes an actuated prismatic joint.

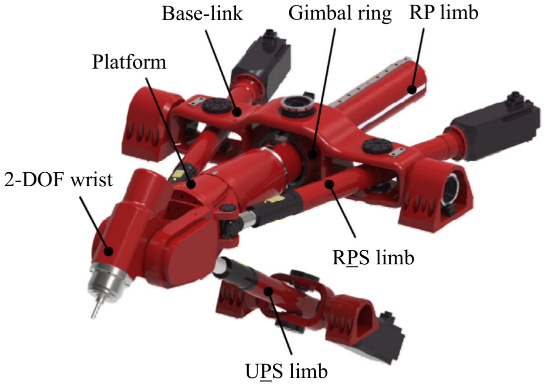

Fig. 1. 3D view of the TriMule robot.

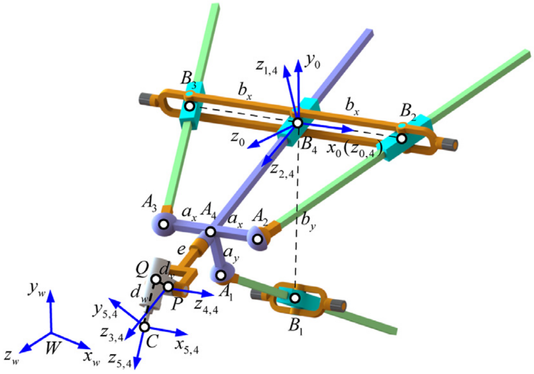

Fig. 2. Schematic diagram of the TriMule robot.

Fig. 2 shows the schematic diagram of the robot. For convenience, we treat universal/spherical joint as two/three revolute joints with the joint axes intersecting at a common point, and we number three actuated limbs as limb 1, 2 and 3, and the passive limb plus the wrist as limb 4. Let $B_i (i = 2, 3, 4)$ be the intersections of the rear R joint axes of limb i with the R joint axis connecting the base link to the machine frame, and B_1 be the center of U joint of limb 1; let $A_i (i = 1, 2, 3)$ be the center of the S joint of limb i, and A_4 be the intersection of the axial axis of the RP limb with its normal plane in which all A_i are placed; and let P be the intersection of two orthogonal axes of the wrist and C be the tool tip. For convenience of inverse displacement analysis, establish the reference frame $\{R_0\}$ with the x_0 axis being the R joint axis connecting the base link to the machine frame, and the z_0 axis being normal to the plane in which all B_i are placed. Meanwhile, establish body fixed frames $\{R_{j,4}\} (j = 0, 1, \cdots, 4)$ with the $z_{j,4}$ axis being the $j + 1$ joint axis, and the tool frame $\{R_{5,4}\}$ with $z_{5,4}$ axis being the spindle axis as shown in Fig. 2. Then, the orientation matrix of $\{R_{5,4}\}$ with respect to $\{R_0\}$ can be expressed by

$$ {}^{0}\boldsymbol{R}_{5,4} = {}^{0}\boldsymbol{R}_{3,4}\,{}^{3,4}\boldsymbol{R}_{5,4} = [\boldsymbol{u} \quad \boldsymbol{v} \quad \boldsymbol{w}] \tag{1} $$

where ${}^{0}\boldsymbol{R}_{3,4}$ and ${}^{3,4}\boldsymbol{R}_{5,4}$ are the orientation matrices of $\{R_{3,4}\}$ with respect to $\{R_0\}$, and that of $\{R_{5,4}\}$ with respect to $\{R_{3,4}\}$, respectively.

$$ {}^{0}\boldsymbol{R}_{3,4} = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ \sin\theta_1 \sin\theta_2 & \cos\theta_1 & -\sin\theta_1\cos\theta_2 \\ -\cos\theta_1 \sin\theta_2 & \sin\theta_1 & \cos\theta_1\cos\theta_2 \end{bmatrix} = [\boldsymbol{s}_{2,4} \times \boldsymbol{s}_{3,4} \quad \boldsymbol{s}_{2,4} \quad \boldsymbol{s}_{3,4}] \tag{2} $$

$$ {}^{3,4}\boldsymbol{R}_{5,4} = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4\cos\theta_5 & \sin\theta_4\sin\theta_5 \\ \sin\theta_4 & \cos\theta_4\cos\theta_5 & -\cos\theta_4\sin\theta_5 \\ 0 & \sin\theta_5 & \cos\theta_5 \end{bmatrix} \tag{3} $$

where $\boldsymbol{s}_{2,4} \times \boldsymbol{s}_{3,4}$, $\boldsymbol{s}_{2,4}$ and $\boldsymbol{s}_{3,4}$ denote the unit vectors of three axes of $\{R_{3,4}\}$; $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ denote those of $\{R_{5,4}\}$; and $\theta_j (j = 1, 2, 4, 5)$ is the rotation angle about the $z_{j-1,4}$ axis of $\{R_{j-1,4}\}$, respectively.

2.2. Inverse displacement analysis

Inverse displacement analysis is concerned with the determination of the actuated joint variables for a given pose represented by the tool tip vector $\boldsymbol{r}_C$ and the tool axis vector $\boldsymbol{w}$. Then, the position vector of P can be expressed as