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Dual-complex k-Fibonacci numbers

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ABSTRACT

In this paper, dual-complex k-Fibonacci numbers are defined. Also, some algebraic properties of dual-complex k-Fibonacci numbers which are connected with dual-complex numbers and k-Fibonacci numbers are investigated. Furthermore, the Honsberger identity, the d'Ocagne's identity, Binet's formula, Cassini's identity, Catalan's identity for these numbers are given.

1. Introduction

Many kinds of generalizations of the Fibonacci sequence have been presented in the literature. In 2007, the k-Fibonacci sequence $\{F_{k,n}\}_{n\in\mathbb{N}}$ is defined by Falcon and Plaza [1] as follows

$$\begin{cases} F_{k,0} &= 0, F_{k,1} = 1 \\ F_{k,n+1} &= kF_{k,n} + F_{k,n-1}, \quad n \geq 1 \\ & or \\ \{F_{k,n}\}_{n\in\mathbb{N}} &= \{0, 1, k, k^2+1, k^3+2k, k^4+3k^2+1, \dots\} \end{cases} \tag{1}$$

Here, k is a positive real number. These sequences were studied by Horadam in [2]. Recently, Falcon and Plaza worked on k-Fibonacci numbers, sequences and matrices in [3]–[4].

In 2010, Bolat and Köse [5] gave properties of k-Fibonacci numbers. In 2014, Catarino [6] obtained some identities for k-Fibonacci numbers. Ramirez[7] defined the k-Fibonacci and the k-Lucas quaternions as follows:

$$D_{k,n} = \{F_{k,n} + iF_{k,n+1} + jF_{k,n+2} + kF_{k,n+3} | F_{k,n}, n-th \text{ k-Fibonacci number}\},$$

and

$$P_{k,n} = \{L_{k,n} + iL_{k,n+1} + jL_{k,n+2} + kL_{k,n+3} | L_{k,n}, n-th \text{ k-Lucas number}\}$$

where i, j, k satisfy the multiplication rules

$$i^2 = j^2 = k^2 = -1, \quad i\,j = -j\,i = k,$$
$$j\,k = -k\,j = i, \quad k\,i = -i\,k = j.$$

In 2015, Polatlı Kızılateş and Kesim [8] defined split k-Fibonacci and split k-Lucas quaternions $(M_{k,\,n})$ and $(N_{k,\,n})$ respectively as follows:

$$M_{k,n} = \{F_{k,n} + iF_{k,n+1} + jF_{k,n+2} + kF_{k,n+3} | F_{k,n}, n-th \text{ k-Fibonacci number}\}$$

where i, j, k are split quaternionic units which satisfy the multiplication rules

$$i^2 = -1, j^2 = k^2 = i\,j\,k = 1, \quad ij = -ji = k,$$
$$jk = -kj = -i, ki = -ik = j.$$

In the 19 th century Clifford invented a new number system by using the notation $(\varepsilon)^2 = 0, \varepsilon \neq 0$. This number system was called dual number system and the dual numbers were represented in the form $A = a + \varepsilon a^*$ with $a, a^* \in \mathbb{R}$ [9]. Afterwards, Kotelnikov (1895) and Study (1903) generalized first applications of dual numbers to mechanics [10],[11]. Besides mechanics, this concept has lots of applications in different areas such as algebraic geometry, kinematics, quaternionic formulation of motion in the theory of relativity. Majernik has introduced the multicomponent number system [12]. There are three types of the four-component number systems which have been constructed by joining the complex, binary and dual two-component numbers. Later, Farid Messelmi has defined the algebraic properties of the dual-complex numbers in the light of this study [13]. There are many applications for the theory of dual-complex numbers. In 2017, the dual-complex Fibonacci numbers defined by Güngör and Azak [14].

Dual-complex numbers [13] w can be expressed in the form as

$$\mathbb{DC} = \{w = z_1 + \varepsilon z_2 | z_1, z_2 \in \mathbb{C} \text{ where } \varepsilon^2 = 0, \varepsilon \neq 0\}. \tag{2}$$

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Table 1
Multiplication scheme of dual-complex numbers.

x	1	i	ε	i ε
1	1	i	ε	i ε
i	i	-,1	i ε	- ε
ε	ε	i ε	0	0
i ε	i ε	- ε	0	0

Here if $z_1 = x_1 + i x_2$ and $z_2 = y_1 + i y_2$, then any dual-complex number can be written

$$w = x_1 + i x_2 + \varepsilon y_1 + i \varepsilon y_2 \tag{3}$$

$i^2 = -1, \varepsilon \neq 0, \varepsilon^2 = 0, (i\varepsilon)^2 = 0.$

The real and dual quaternions form a division algebra, but dual-complex numbers form a commutative ring with characteristics 0. Moreover, the multiplication of these numbers gives the dual-complex numbers the structure of 2-dimensional complex Clifford Algebra and 4-dimensional real Clifford Algebra. The base elements of the dual-complex numbers satisfy the following commutative multiplication scheme (Table 1).

Five different conjugations can operate on dual-complex numbers [13] as follows:

$$\begin{aligned} w &= z_1 + \varepsilon z_2 = x_1 + i x_2 + \varepsilon y_1 + i\varepsilon y_2, z_2 \neq 0, \\ w^{*_1} &= (x_1 - i x_2) + \varepsilon(y_1 - i y_2) = (z_1)^* + \varepsilon(z_2)^*, \\ w^{*_2} &= (x_1 + i x_2) - \varepsilon(y_1 + i y_2) = z_1 - \varepsilon z_2, \\ w^{*_3} &= (x_1 - i x_2) - \varepsilon(y_1 - i y_2) = z_1^* - \varepsilon z_2^*, \\ w^{*_4} &= (x_1 - i x_2)(1 - \varepsilon \frac{y_1 + i y_2}{x_1 + i x_2}) = (z_1)^*(1 - \varepsilon \frac{z_2}{z_1}), \\ w^{*_5} &= (y_1 + i y_2) - \varepsilon(x_1 + i x_2) = z_2 - \varepsilon z_1. \end{aligned} \tag{4}$$

Therefore, the norm of the dual-complex numbers is defined as

$$\begin{aligned} N_w^{*_1} &= \|w \times w^{*_1}\| = \sqrt{\left|z_1^2\right| + 2\varepsilon Re(z_1 z_2{}^*)}, \\ N_w^{*_2} &= \|w \times w^{*_2}\| = \sqrt{z_1^2}, \\ N_w^{*_3} &= \|w \times w^{*_3}\| = \sqrt{\left|z_1^2\right| - 2i\varepsilon Im(z_1 z_2{}^*)}, \\ N_w^{*_4} &= \|w \times w^{*_4}\| = \sqrt{\left|z_1^2\right|}, \\ N_w^{*_5} &= \|w \times w^{*_5}\| = \sqrt{z_1 z_2 + \varepsilon(z_2^2 - z_1{}^2)}. \end{aligned} \tag{5}$$

Similarly, in 2017, the dual-complex Fibonacci and Lucas numbers defined by Güngör and Azak [14] as follows

$$\begin{aligned} \mathbb{DC}F_n &= (F_n + iF_{n+1}) + \varepsilon(F_{n+2} + iF_{n+3}) \\ &= F_n + iF_{n+1} + \varepsilon F_{n+2} + i\varepsilon F_{n+3} \end{aligned} \tag{6}$$

and

$$\begin{aligned} \mathbb{DC}L_n &= (L_n + iL_{n+1}) + \varepsilon(L_{n+2} + iL_{n+3}) \\ &= L_n + iL_{n+1} + \varepsilon L_{n+2} + i\varepsilon L_{n+3} \end{aligned} \tag{7}$$

where the basis $\{1, i, \varepsilon, i\varepsilon\}$ satisfy the conditions

$i^2 = -1, \varepsilon \neq 0, \varepsilon^2 = 0, (i\varepsilon)^2 = 0.$

In this paper, the dual-complex k-Fibonacci numbers and the dual-complex k-Lucas numbers will be defined respectively, as follows

$$\mathbb{DC}F_{k,n} = F_{k,n} + iF_{k,n+1} + \varepsilon F_{k,n+2} + i\varepsilon F_{k,n+3},$$

and

$$\mathbb{DC}L_{k,n} = L_{k,n} + iL_{k,n+1} + \varepsilon L_{k,n+2} + i\varepsilon L_{k,n+3},$$

$i^2 = -1, \varepsilon \neq 0, \varepsilon^2 = 0, (i\varepsilon)^2 = 0.$

where $F_{k,\, n}$, nth k-Fibonacci number and $L_{k,\, n}$, nth k-Lucas number.

The aim of this work is to present in a unified manner a variety of algebraic properties of the dual-complex k-Fibonacci numbers as well as both the dual-complex numbers and k-Fibonacci numbers. In particular, using five types of conjugations, all the properties established for dual-complex numbers are also given for the dual-complex k-Fibonacci numbers. In addition, Binet's Formula, the Honsberger identity, the d'Ocagne's identity, Cassini's identity and Catalan's identity for these numbers are given.

2. The dual-complex k-Fibonacci numbers

The dual-complex k-Fibonacci and k-Lucas numbers can be define by with the basis $\{1, i, \varepsilon, i\varepsilon\}$, where i, ε and $i\varepsilon$ satisfy the conditions

$i^2 = -1, \varepsilon \neq 0, \varepsilon^2 = 0, (i\varepsilon)^2 = 0.$

as follows

$$\begin{aligned} \mathbb{DC}F_{k,n} &= (F_{k,n} + iF_{k,n+1}) + \varepsilon(F_{k,n+2} + iF_{k,n+3}) \\ &= F_{k,n} + iF_{k,n+1} + \varepsilon F_{k,n+2} + i\varepsilon F_{k,n+3} \end{aligned} \tag{8}$$

and

$$\begin{aligned} \mathbb{DC}L_{k,n} &= (L_{k,n} + iL_{k,n+1}) + \varepsilon(L_{k,n+2} + iL_{k,n+3}) \\ &= L_{k,n} + iL_{k,n+1} + \varepsilon L_{k,n+2} + i\varepsilon L_{k,n+3}. \end{aligned} \tag{9}$$

With the addition and multiplication by real scalars of two dual-complex k-Fibonacci numbers, the dual-complex k-Fibonacci number can be obtained again.

Then, the addition and subtraction of the dual-complex k-Fibonacci numbers are defined by

$$\begin{aligned} \mathbb{DC}F_{k,n} \pm \mathbb{DC}F_{k,m} &= (F_{k,n} + iF_{k,n+1} + \varepsilon F_{k,n+2} + i\varepsilon F_{k,n+3}) \\ &\quad \pm (F_{k,m} + iF_{k,m+1} + \varepsilon F_{k,m+2} + i\varepsilon F_{k,m+3}) \\ &= (F_{k,n} \pm F_{k,m}) + i(F_{k,n+1} \pm F_{k,m+1}) \\ &\quad + \varepsilon(F_{k,n+2} \pm F_{k,m+2}) + i\varepsilon(F_{k,n+3} \pm F_{k,m+3}) \end{aligned} \tag{10}$$

The multiplication of a dual-complex k-Fibonacci number by the real scalar λ is defined as

$$\lambda\mathbb{DC}F_{k,n} = \lambda F_{k,n} + i\lambda F_{k,n+1} + \varepsilon\lambda F_{k,n+2} + i\varepsilon\lambda F_{k,n+3}. \tag{11}$$

The multiplication of two dual-complex Fibonacci numbers is defined by

$$\begin{aligned} \mathbb{DC}F_{k,n} \times \mathbb{DC}F_{k,m} &= (F_{k,n} + iF_{k,n+1} + \varepsilon F_{k,n+2} + i\varepsilon F_{k,n+3}) \\ &\quad (F_{k,m} + iF_{k,m+1} + \varepsilon F_{k,m+2} + i\varepsilon F_{k,m+3}) \\ &= (F_{k,n}\ F_{k,m} - F_{k,n+1}\ F_{k,m+1}) \\ &\quad + i(F_{k,n+1}\ F_{k,m} + F_{k,n}\ F_{k,m+1}) \\ &\quad + \varepsilon(F_{k,n}F_{k,m+2} - F_{k,n+1}F_{k,m+3} \\ &\quad + F_{k,n+2}F_{k,m} - F_{k,n+3}F_{k,m+1}) \\ &\quad + i\varepsilon(F_{k,n+1}F_{k,m+2} + F_{k,n}F_{k,m+3} \\ &\quad + F_{k,n+3}F_{k,m} + F_{k,n+2}F_{k,m+1}) \\ &= \mathbb{DC}F_{k,m} \times \mathbb{DC}F_{k,n}. \end{aligned} \tag{12}$$

Also,the dual-complex k-Fibonacci numbers provide the properties of (4)–(5) [13].

Five kinds of conjugation can be defined for dual-complex numbers [13]. Therefore, conjugation of the dual-complex k-Fibonacci number is defined in five different ways as follows

$$\mathbb{DC}F_{k,n}{}^{*_1} = F_{k,n} - iF_{k,n+1} + \varepsilon F_{k,n+2} - i\varepsilon F_{k,n+3}, complex - conjugation \tag{13}$$