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Switching induced oscillations in discrete one-dimensional systems

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ABSTRACT

In ecological modeling, seasonality can be represented as an alternation between environmental conditions. We consider a switching strategy that alternates between two undesirable dynamics and find that they can yield a desirable periodic behavior in the case of the Beverton–Holt, Ricker, and modified Ricker maps, which have been extensively used to model ecological populations. For the Ricker and modified Ricker models, we observe coexistence of attractors, which, under the same conditions, define basin of attractions, and the final dynamic behavior depends on the initial conditions.

1. Introduction

Over the years, theoretical ecologists have modeled population dynamics using either discrete or continuous equation methods. In the former case, maps have been the method of choice [1–6]. In particular, the logistic map has played a central role in the development and understanding of complex dynamic systems [7]. Originally, the logistic map has been used to study populations of non-overlapping generations, and is represented by the following relation between the new generation (X_{n+1}) and the old generation (X_n).

$$X_{n+1} \;=\; f_C(X_n) \;=\; C\, X_n(1-X_n) \tag{1}$$

Independently from ecological studies, for the last ten years, alternate dynamics strategies have been the center of attention due to the so-called Parrondo paradox [8–10], where two losing games can be combined to yield a winning game. Furthermore, the idea that "lose + lose = win" has been extended to "chaos + chaos = periodic" in one-dimensional maps [11–19]. In an extension of the so-called Parrondian games, we have analyzed the dynamics of the logistic map, where we represent two seasons by alternating two relevant parameter values. We have considered the case where the alternation of undesirable dynamic behaviors yield a desirable behavior in the context of seasonality in the logistic map, where we alternate the parameter values between even and odd iterations. For example, the alternation between a parameter that would drive the logistic map to extinction, and a parameter that would drive the logistic map to chaos yields stable oscillations. So, in the context of population dynamics, we have considered cases where "undesirable + undesirable = desirable" dynamic behaviors occur as a result of a simple alternation of parameters [17–19].

In our present discussion, we extend our simple seasonality modeling strategy to several one-dimensional ecologically relevant maps and find that the "undesirable + undesirable = desirable", as well as the "chaos + chaos = periodic" behaviors are not unique to the logistic map. In Section 2, we consider a generalization of the Beverton–Holt map [20], and in Section 3, we analyze one of the most popular limits of this map, which some authors sometimes refer to as the Ricker map [20]. In Section 4, we consider three modified Ricker models [21], and, in Section 5, we discuss and summarize our results.

2. Modified Beverton–Holt model

The Beverton–Holt map has been used to model fish population dynamics, and it is considered as a useful map by ecologists [20,21]. In our analysis we start with the dimensionless map:

$$X_{n+1} \;=\; \frac{C\, X_n\, Exp[-X_n]}{(1+b(1-Exp[-X_n]))} \tag{2}$$

where C is the most relevant parameter because larger values of b tend to stabilize the map's dynamics. In our case, we consider three b values : 0, 1/2, and 1. In particular, $b=0$ yields a simpler map, which some authors identify as the Ricker Model, which we consider in Section 3.

In our analysis, we start by constructing bifurcation diagrams for Eq. (2) and by identifying the parameter values associated with undesirable (extinction or chaos) dynamical behaviors. For example, in Figs. 1 and 2, we depict the bifurcation diagram for $b=1/2$ and $b=1$. From these figures, we notice that as we increase the

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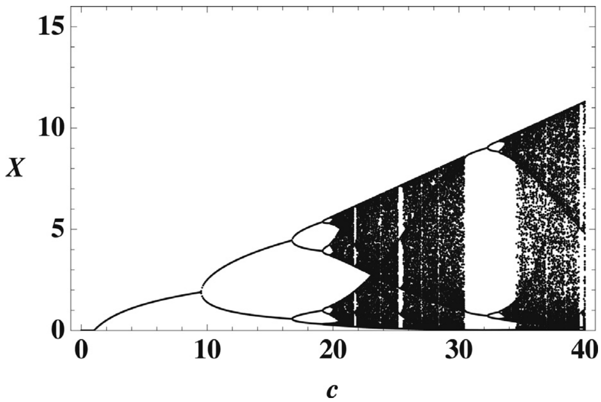

Fig. 1. Modified Beverton–Holt Model for $b = 1/2$.

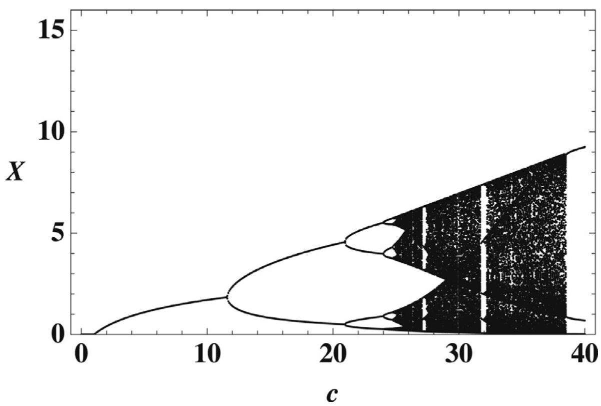

Fig. 2. Modified Beverton–Holt Model for b = 1.0.

value of b, the C parameter values related to chaotic dynamics shift towards larger values.

In the case of $b = 1/2$, only values of $C > 20$ may show chaotic dynamics. In contrast, for $b = 1$, the chaotic behavior occurs for C values greater than twenty four. Since we are interested in population extinction, we depict the bifurcation diagram for values of C less than twelve, where it is clear that for $C \leq 1$, extinction is the only stable and physical solution. Hence, values of C less than or equal to unity imply an undesirable dynamical behavior.

Next, we alternate the parameter values between even and odd iterations:

$$X_{n+1} = \begin{cases} f_{C_e}(X_n) = \frac{C_e\ X_n\ Exp[-X_n]}{(1+b(1-Exp[-X_n]))} & \text{if } n \text{ even} \\ f_{C_o}(X_n) = \frac{C_o\ X_n\ Exp[-X_n]}{(1+b(1-Exp[-X_n]))} & \text{if } n \text{ odd} \end{cases} \quad (3)$$

From Figs. 1 and 2 we can select values associated with chaotic behavior for C_o, and use C_e as a bifurcation parameter. From the resulting diagram, we can easily identify parameter values that, when alternated with C_o, yield stable oscillations. For example, if we consider $C_o = 29$, which, as seen from Fig. 1, yields chaotic trajectories, Eq. (2) yields a bifurcation diagram depicted in Fig. 3.

From Fig. 4, we observe stable oscillations for values less than unity, and for values between 36.25 and 37. In the former, case the values are associated with extinction, while in the latter case, the values are associated with chaotic trajectories if we use Eq. (3). Therefore, in one case, we have an example of "undesirable + undesirable = desirable" while in the other case we observe that "chaos + chaos = periodic." In contrast to other approaches, we construct bifurcation diagrams to find parameter values that, when alternated, yield stable trajectories. Our approach is straightforward for discrete systems and yields intervals of valid parameter values rather than single values. A more dramatic example of "chaos + chaos = periodic" occurs for $b = 1/2$, $C_o = 20$, and C_e between 24 and 40, as depicted in Fig. 5, as compared with Fig. 1, although we still observe oscillations for values of C_e less than unity.

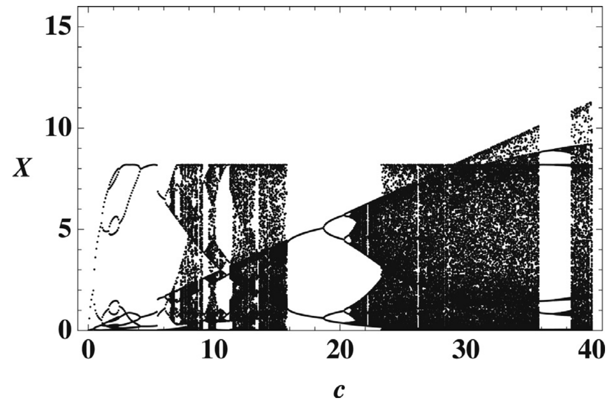

Fig. 3. Bifurcation diagram as a function of C_e, with $C_o = 29$, and $b = 1/2$ for the Beverton–Holt map.

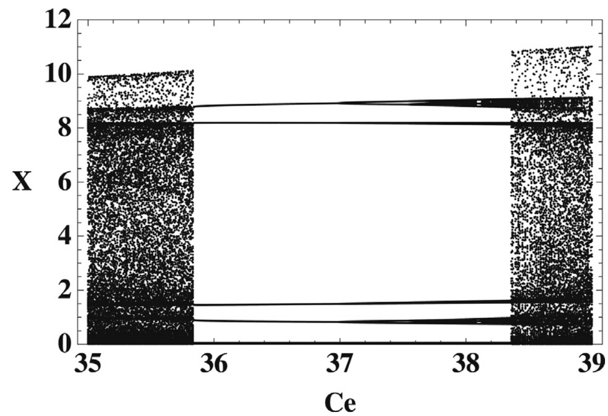

Fig. 4. Bifurcation diagram as a function of C_e, with $C_o = 29$, and $b = 1/2$ for the Beverton–Holt map.

For completeness, we depict in Fig. 6, bifurcation diagrams for $b = 1$ and $C_o = 28$. Comparing Figs. 2 and 6, we find that values between 38 and 38.5 yield aperiodic oscillations when used in Eq. (2), but, when used with Eq. (3), the same values yield stable oscillations, which is another example of "'chaos + chaos = periodic". Finally, from Fig. 2, we consider $C_o = 39$, which yields stable oscillations when using Eq. (3). In Fig. 7, we depict the bifurcation diagrams for $C_o = 39$, and $b = 1$. In this case, we want to focus on the interval $0 < C_e < 12$. If we compare Figs. 2 and 7, the effect of alternate dynamics yields chaotic behavior around $C_e = 2$. Therefore, for $C_o = 39$ and $C_o \approx 2$, we have a case of "periodic + periodic = chaos," another example of this case occurs for $b = 1/2$, $C_o = 31.8$ and C_e between 7 and 16.

In the limiting case, $b = 0$, we recover a map referred to as the Ricker map:

$$X_{n+1} = \begin{cases} f_{C_e}(X_n) = C_e\ X_n\ Exp[-X_n] & \text{if } n \text{ even} \\ f_{C_o}(X_n) = C_o\ X_n\ Exp[-X_n] & \text{if } n \text{ odd} \end{cases} \quad (4)$$

In Fig. 8, we depict the bifurcation diagram for Eq. (4) for $C_o = C_e$, which can be compared with the diagrams in Figs. 1 and 2.

As before, we pick a C_o value associated with a chaotic trajectory and alternate with C_e, which we use as the bifurcation parameter, as illustrated in Fig. 9. From the bifurcation diagram, we can easily pick values of $C_e < 1$ that correspond to "undesirable + undesirable = desirable" dynamics to model seasonality. We can also