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Multiple soliton solutions of the nonlinear partial differential equations describing the wave propagation in nonlinear low–pass electrical transmission lines

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ABSTRACT

This paper focuses on investigating soliton and other solutions using three integration schemes to integrate a nonlinear partial differential equation describing the wave propagation in nonlinear low–pass electrical transmission lines. By applying the Kirchhoff's laws and complex transformation, the nonlinear low–pass electrical transmission lines are converted into an equation wave propagation in nonlinear ODE low–pass electrical transmission lines. Later on, mentioned integration schemes viz modified Kudryashov method, sine–Gordon equation expansion method and extended sinh–Gordon equation expansion method are used to carry out new hyperbolic and trigonometric solutions which shows the consistency via computerized symbolic computation package maple. Various types of solitary wave solutions are derived including kink, anti-kink, dark, bright, dark-bright, singular, combined singular, and periodic singular wave soliton solutions. The corresponding three integration schemes are robust and effective for acquiring the new dark, bright, dark–bright, singular or combined singular and optical soliton solutions of the wave propagation in nonlinear low–pass electrical transmission lines. To show the real physical significance of the studied equation, some three dimensional (3D) and two dimensional (2D) figures of obtained solutions are plotted with the use of the Matlab software under the proper choice of arbitrary parameters. Moreover, all derived solutions were verified back into its corresponding equation with the aid of maple program.

1. Introduction

The nonlinear evolution equations (NLEEs) describe physical phenomena in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibres, biology, solid-state physics, etc. Exact solutions of NLEEs play an important role in the proper understanding of mechanisms of many physical phenomena and processes in various areas of natural sciences. Moreover, the effort in investigating exact solitary wave solutions of NLEEs by means of several distinct schemes has grown rapidly in recent years which is one of the most exciting and advanced topic of nonlinear science, theoretical physics and engineering. Thus, it is very important to know the theory of the special waves namely solitons.

Solitons play a pivotal role in many physical systems and they appear in various form such as kink, pulse, envelope, bright, breather, dark and many others. Soliton is a localized wave form that travels along the system with constant velocity and undeformed shape. Physicists and Engineers have proved that solitons are extremely interesting due to their localized and stable nature in applied field like nonlinear optics, plasma physics, communications and electronic engineering, fluid mechanics, ocean engineering, signal processing and so on. In recent past, there are several distinct influential integration schemes have introduced and implemented for investigating the exact and approximated solutions of these NLEEs, such as the $\frac{G'}{G}$-expansion method, extended $\frac{G'}{G}$-expansion method, auxiliary equation method, new auxiliary equation method, new Jacobi elliptic function expansion method, modified Kudryashov method, the improved extended

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tanh–function method, the generalized Riccati equation mapping method, the sine–Gordon equation expansion method, the extended sinh–Gordon equation method, exp function method, semi-inverse variational principle, modified simple equation method, and many more [1–36].

Under the investigation of solitary wave solutions, we consider the nonlinear PDE describing the wave propagation in nonlinear low–pass electrical transmission lines [2–6]:

$$\frac{\delta^2 V(x,t)}{\delta t^2}+\alpha\frac{\delta^2 V^2(x,t)}{\delta t^2}-\beta\frac{\delta^2 V^3(x,t)}{\delta t^2}+\delta^2\frac{\delta^2 V^2(x,t)}{\delta x^2}+\frac{\delta^4}{12}\frac{\delta^4 V(x,t)}{\delta x^4}=0. \tag{1}$$

where $V(x,\ t)$ is the voltage of the transmission lines, and $\alpha,\ \beta,\ \delta$ are all constants. The variable x is interpreted as the propagation distance and t is the slow time. The physical details of the derivation of Eq. (1) using the Kirchhoff's laws are given in [2], which are omitted here for the sake of shortness.

The study of nonlinear electrical transmission lines (NLTLs) and its exact solutions is important for various applied fields such as connecting radio transmitters and receivers with their antennas, distributing cable television signals, trunk lines routing calls between telephone switching centers, computer network connections and high-speed computer data buses [1–16]. Moreover, in communications and electronic engineering, a transmission line is a specialized medium or other structure designed to carry alternating current of radio frequency [1]. NLTLs is also provide a useful way to check how the nonlinear excitations behave inside the nonlinear medium and to model the exotic properties of new systems [9–14].

Many integration schemes including auxiliary equation method, new Jacobi elliptic function expansion method, the Kudryashov method, the $\frac{G'}{G}$-expansion method, the improved extended tanh-function method and the generalized Riccati equation mapping method were employed to derive explicit solitary wave solutions of the electrical transmission lines equation (NETLEs) in the past [2–6]. The NETLEs are very convenient tools for the study of propagation of electrical solitons which can propagate in the form of voltage waves in nonlinear dispersive media.

The main aim of this study is to explore new dark, bright, dark–bright, singular or combined singular, optical soliton and other solutions of the wave propagation in nonlinear low–pass electrical transmission lines using three efficient distinct integration schemes, known as the modified Kudryashov method, the sine–Gordon equation expansion method and the extended sinh–Gordon equation expansion method.

The residue of the paper is organized as follows. A brief discussion modified Kudryashov method and its applications are presented in Section 2. Section 3 and its sub-sections deal with the description of the sine-Gordon equation and its application. Algorithm of the sinh-Gordon equation expansion method and its application are also discussed in Section 4. In Section 5, we compare the solutions obtained in this paper with other known solutions. Finally, we draw a concluding remark about the studied methods and presented results in Section 5.

2. Modified Kudryashov method for solving the wave propagation in nonlinear low-pass electrical transmission lines equation

The modified Kudryashov method [19–21] is deliberated as a new robust problem–solving technique that has received significant attention to explore new exact solutions of nonlinear differential equations executed in mathematical physics and other applied fields.

2.1. Key ideas of the modified Kudryashov method

We present a succinct about the modified Kudryashov method to generate new exact solutions for a given nonlinear partial differential equation. In this regard, we consider a general form of nonlinear partial differential equation as

$$P(V,V_x,V_t,V_{xx},V_{xt},V_{tt},\dots)=0. \tag{2}$$

where the function $V=V(x,t)$ is unknown and P is a polynomial function with respect to some functions or specified variables, which contains nonlinear terms and highest order derivatives of the $V(x,\ t)$. The main steps are as follows:

Step-1: Introducing the transformation $V(x,t)=V(\xi)$where $\xi=\sqrt{k}(x-vt)$, converts Eq. (2) to the following nonlinear ordinary differential equation

$$R(V,V',V'',\dots)=0. \tag{3}$$

where, R is a polynomial of V and its derivatives and the superscripts suggest the ordinary derivatives with respect to ξ.

Step-2: It is supposed that the solution $U(\xi)$of the nonlinear Eq. (3) can be presented as

$$V(\xi)=a_0+\sum_{i=1}^{N}a_iQ^i(\xi). \tag{4}$$

where the arbitrary constants $a_i(i=1,2,\dots,N)$ are determined latter but $a_N\neq 0$ and is a positive integer, N which is determined by using balancing principle on Eq. (4) and satisfies the following an ansatz equation

$$Q'(\xi)=\left(Q^2(\xi)-Q(\xi)\right)\ln(a). \tag{5}$$

where, $a\neq 0,1$ and the general solution of the Eq. (5) is

$$Q(\xi)=\frac{1}{1+da^{\xi}}.$$

Step 3: By inserting Eq. (4) along with Eq. (5) into Eq. (3) and equating the coefficients of powers of $Q^i(\xi)$ to zero, we receive a system of algebraic equations. Solving these system, we secure the value of free parameters a_0,a_1, k and v. After, putting the obtained values in Eq. (4), finally generates new exact solutions for the Eq. (2).

2.2. Exact solution of the nonlinear low–pass electrical transmission lines

By considering the traveling wave transformation as:

$$V(x,t)=V(\xi),\ \ \xi=\sqrt{k}(x-vt) \tag{6}$$

where k and v are arbitrary constant to be determined later. Under the transformation Eq. (6), the wave propagation nonlinear low–pass electrical transmission lines Eq. (1) can be changed to a following nonlinear ODE:

$$k\delta^4V''+12\left(\delta^2-v^2\right)V+12\alpha v^2V^2-12\beta v^2V^3=0. \tag{7}$$

Now balancing between V'' and V^3, we obtain $N=1$. Then, assuming solution of the Eq. (7) is

$$V(\xi)=a_0+a_1Q(\xi). \tag{8}$$

By inserting Eq. (8) along with its first and second derivatives into Eq. (7) and comparing the terms in the resulting equation, a nonlinear system is gained which by solving it, we determined the following sets:

Set 1: $a_0=0$, $a_1=\frac{2}{3}\frac{\alpha}{\beta}$, $k=-\frac{24\alpha^2}{(2\alpha^2-9\beta)\delta^2(ln(a))^2}$, and $v=\pm 3\delta\sqrt{\frac{-\beta}{2\alpha^2-9\beta}}$.