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Hybrid modeling of quasi-particles: Algebra, Fock space and condensation

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ABSTRACT

In this study, we first introduce a new quasi-particle algebra, which enables us to effectively describe a unified framework for both bosons and fermions. We then study general thermodynamical and statistical properties of a hybrid-type gas model of these quasi-particles. In this context, we specifically focus on the conditions under which the hybrid-type quasi-particle gas condensation would occur in the present model. The results obtained in this work reveal that the present gas model of quasi-particles can be used to approximate non-linear behavior observed in composite particle systems such as in studies on the phenomenon of high-T_c superconductivity in a given material.

1. Introduction

To understand the details about the nature of many-body interacting physical systems have often required considering non-standard approaches such that either non-linearity or non-ideality factors in such systems can be dealed with the properties of a non-standard quantum formalism. One of the effective ways to approximate such non-linearities observed in complex systems is to use some quantum deformed model. Deformed Bose and Fermi gas models based on their respective deformed bosonic and fermionic oscillator algebras have recently been provided developments in several areas of research in many aspects. Historically, the q-deformed bosonic oscillator algebra was first introduced by Arik and Coon [1] and lately accomplished by Macfarlane [2] and Biedenharn [3] by means of some elements of Jackson's q-calculus [4]. In this context, it should be mentioned that a two-parameter generalization of the deformed bosonic oscillator algebra was introduced in [5] and also, one possible generalization of the q-calculus has been carried out by the Fibonacci calculus based on the bosonic Fibonacci oscillator algebras [6,7].

As is mentioned above, deformed Bose and Fermi gas models have been applied to a wide spectrum of research covering from cosmology to quantum computation. For instance, based on the idea that the extremal quantum black holes obey the deformed statistics [8], some cosmological problems related to the dark constituents of universe have been addressed by some researchers such as the ones with deformed dark energy [9,10] and dark matter [11,12]. Meanwhile, in the framework of thermodynamic geometry, the deformed Bose and Fermi gas models have been used to analyze both the properties of intermediate-statistics [13,14] and the condensation characteristics [15,16]. Also, possible connections between deformed quantum oscillators and the fractional statistics have been investigated [17–22]. One of the main advantageous of these deformed models is on the possibility to obtain new clues about the details of both interactions of particles and their compositeness. In this framework, it should be mentioned that composite particles (or quasi-particles) are realizable by deformed oscillators [23,24], and the entanglement characteristics in these systems can also be tackled by some deformation structure function [25]. Moreover, as has been recently discussed in the works of [26,27] for the thermal and electrical properties of a solid, the q-deformation acts as a factor of impurity or disorder in a given material and hence, any deviation between theory and experiment can be controlled with an appropriate deformed (quasi)particle model. Although, general thermostatistical properties of deformed boson and fermion systems have been investigated to some extent in the literature [28–40], their consequences and potential applications are still under active investigation.

In the present work, our aim is to introduce a different quasi-particle algebra, which serves as a unification of quantum field-theoretical analysis of both bosons and fermions. The quasi-particles having both bosonic and fermionic features can be called as hybrid-type quasi-particles or hybrid(e)ons, and the

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corresponding gas model of these quasi-particles will be examined within the framework of statistical mechanics.

The paper is organized as follows: In Section 2, we present the quasi-particle algebra called the hybrid-type quasi-particle algebra, and give its Fock space properties. Also, we extend our formalism to the multi-mode case of these quasi-particles, and particularly present the intercept for the two-particle correlation function. In Section 3, general thermodynamical and statistical properties of a gas model of hybrid-type quantum oscillators are analyzed, and particular emphasis is given to the effect of deformation on the conditions under which the hybrid-type quasi-particle gas condensation would occur in such a system. Finally, we will give our concluding remarks as well as a discussion on possible physical applications of the present model in the last section.

2. The hybrid-type quasi-particle algebra and its multi-mode case

In this section, we wish to present the algebraic properties of the model under consideration. The model algebra is defined by the following relations:

$$[a, a^*] = r + s\,(-1)^{\hat{N}}, \quad [\hat{N}, a] = -a, \quad [\hat{N}, a^*] = a^*, \tag{1}$$

where $0 \leq (r, s) \leq 1$ and $r + s = 1$. This algebra interpolates the usual boson and fermion quantum algebras. Therefore, we will call a (quasi)particle obeying the above algebra as the hybrid-type quasi-particle, which differs from the anyon [41–43]. Here, we have assumed the following proposition in the model: All quantum particles have fermionic property as well as bosonic property, and they obey the algebra in Eq. (1). For $r = 1$ and $s = 0$, the union algebra reduces to the usual boson algebra, whereas it reduces to the usual fermion algebra in the limit $r = 0$ and $s = 1$. Hence, we can remark that the situation dictated by the model algebra in Eq. (1) bears a resemblance to the wave-particle duality in quantum world, which states that all particles have wave property and all waves also have particle property. Due to the dual characters of a particle obeying the algebra in Eq. (1), we can call such a physical situation as the fermion-boson duality.

Since we have the condition $r + s = 1$, the algebra in Eq. (1) has essentially just one deformation parameter. Therefore considering $r = q$, we rewrite the model algebra in Eq. (1) as

$$[a, a^*] = q + (1-q)\,(-1)^{\hat{N}}, \quad [\hat{N}, a] = -a, \quad [\hat{N}, a^*] = a^*, \tag{2}$$

where $0 \leq q \leq 1$. This algebra has two limiting cases: It reduces to the usual boson algebra in the limit $q = 1$, while it reduces to the usual fermion algebra in the limit $q = 0$. Thus, for $0 < q < 1$, this algebra describes the quantum particles possessing both fermionic property and bosonic property. For the model algebra in Eq. (2), we have the following deformed number operator definition:

$$a^* a = q\,\hat{N} + \frac{(1-q)}{2}\,[1 - (-1)^{\hat{N}}], \tag{3}$$

which gives $\hat{N} = a^* a$ when $q = 1$, whereas it gives $a^* a = (1/2)[1 - (-1)^{\hat{N}}]$ when $q = 0$. In the limiting case of $q = 0$, we have the two-dimensional Fock space, so we can identify the term $(1/2)[1 - (-1)^{\hat{N}}]$ with $\hat{N}$, which also corresponds to $a^* a = \hat{N}$ for the usual fermion case. Hence, the action of $\hat{N}$ is standard in the sense that

$$\hat{N}\,|n\rangle = n\,|n\rangle, \quad n = 0, 1, 2, ..., n_{\max}, \tag{4}$$

where the meaning of $n_{\max}$ is given below, and also the actions of the remaining operators are given by

$$a\,|n\rangle = \sqrt{\{n\}_q}\,|n-1\rangle, \tag{5}$$

$$a^*\,|n\rangle = \sqrt{\{n+1\}_q}\,|n+1\rangle, \tag{6}$$

where the deformed number $\{n\}_q$ is defined as

$$\{n\} \equiv \{n\}_q = q\,n + \frac{(1-q)}{2}\,[1 - (-1)^n]. \tag{7}$$

One can easily check that the deformed number $\{n\}_q$ reduces to n in the bosonic limit $q = 1$, while it reduces to the result $(1/2)[1 - (-1)^n]$ in the fermionic limit $q = 0$. Since we have $\{2\}_q = 0$ in the fermionic limit, we then have $n_{\max} = 1$ for $q = 0$. But, unless $q = 0$, we have the infinite-dimensional Fock space along with $n_{\max} = \infty$. In such a case, the deformed number involves the alternating term, so we have

$$\{2m\}_q = 2q\,m, \quad \{2m+1\}_q = 2q\,m + 1, \quad m = 0, 1, 2, ... \tag{8}$$

which states that all number eigenstates have positive norm for the interval $0 \leq q \leq 1$. Therefore, we will consider this interval in the rest of the calculations in this work.

Now, let us consider the following Hamiltonian in the canonical ensemble for a hybrid-type gas model of non-interacting quasi-particles:

$$\hat{H} = \omega\,\hat{N}, \tag{9}$$

where we set $\hbar = 1$. The energy eigenvalue is then given by $\varepsilon_n = \omega\,n$. Hence, following the standard prescription in statistical mechanics [44–46] and using Eqs. (2)–(7), we find out the statistical distribution function for the hybrid-type gas model of quasi-particles as follows:

$$< n > \equiv < n >_q = q < n >_{BE} + (1-q) < n >_{FD} = \frac{q}{e^{\beta\omega} - 1} + \frac{1-q}{e^{\beta\omega} + 1}, \tag{10}$$

where $\beta = 1/kT$ with the Boltzmann constant k and the temperature T. The relation in Eq. (10) reduces to the Fermi-Dirac (FD) statistics $< n >_{FD}$ in the fermionic limit $q = 0$, whereas it reduces to the Bose-Einstein (BE) statistics $< n >_{BE}$ in the bosonic limit $q = 1$.

Besides, the hybrid-type quasi-particle algebra given in Eq. (2) can be extended into the multi-mode case as follows:

$$[a_i, a_j^*] = \delta_{ij}\,[q + (1-q)\,(-1)^{\hat{N}_i}], \tag{11}$$

$$a_i a_j = (2q-1)\,(1-\delta_{ij}) a_j\,a_i, \tag{12}$$

$$a_i^* a_j^* = (2q-1)\,(1-\delta_{ij}) a_j^*\,a_i^*, \tag{13}$$

and also,

$$a_i^* a_i = q\,\hat{N}_i + \frac{(1-q)}{2}\,[1 - (-1)^{\hat{N}_i}]. \tag{14}$$

From the above relations, the two-particle momentum correlation function for coinciding modes is derived as

$$< a_i^* a_i^* a_i a_i > = \frac{2\,q\,(-1 + e^{\beta\omega} + 2q)}{(-1 + e^{\beta\omega})^2\,(1 + e^{\beta\omega})}, \tag{15}$$

and also, we have the result

$$< a_i^* a_i > = \frac{(-1 + e^{\beta\omega} + 2q)}{(-1 + e^{\beta\omega})\,(1 + e^{\beta\omega})},$$

where the bracket $< ... >$ stands for the statistical (thermal) average, and we have omitted the fixed momentum index $\vec{k}$ for the sake of simplicity. Thus, we can compute the q-deformed second order intercept $\lambda_q^{(2)}$ for the hybrid-type quasi-particle gas model defined as

$$\lambda_q^{(2)} = \frac{< (a^*)^2 (a)^2 >}{< a^* a >^2} - 1, \tag{16}$$