Chaos, Solitons and Fractals 115 (2018) 96–107

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Unknown input observer design for a class of fractional order nonlinear systems

Vivek Sharma*, Manoj Shukla, B.B. Sharma

Dapartment of Electrical Engineering, NIT Hamirpur, Himachal Pradesh, India

ARTICLE INFO

Article history:
Received 1 January 2018
Revised 16 August 2018
Accepted 20 August 2018

Keywords:
Observer design
Imperfect systems
Fractional order nonlinear systems
Differential Mean Value (DMV) Theorem

ABSTRACT

Analysis and control of fractional order (FO) nonlinear systems is a challenging problem. In earlier works, as highlighted in literature, stability conditions for the FO LTI systems are analytically derived and these results are extended to formulate LMI conditions to express the stability of the FO LTI systems. In present work, design of full order and reduced order observers for imperfect fractional order nonlinear systems is presented. Imperfections in real system are silent dynamics and can be modeled as unknown input. To design observer for such system, unknown input observer (UIO) design concepts are used and LMI conditions for the existence of observer are analytically derived. For this purpose, Differential Mean Value (DMV) theorem is used and nonlinear term in the error dynamics is alternatively expressed in appropriate equivalent form. As a result, error dynamics evolves as Linear Parameter Varying (LPV) system and then stability results for FO LTI systems are extended to stabilize FO nonlinear error dynamical systems. LMI conditions for the existence of unknown input observer for the two cases $0<\alpha<1$ and $1<\alpha<2$ are analytically derived. Feasible solution of LMI gives the observer design matrices directly. Finally, results of simulation are presented to authenticate the proposed approach.

1. Introduction

The idea of fractional calculus is as old as the integer order calculus [1]. In fractional order systems, the order α of the system is a real number in the range (0, 2). For integer order LTI system complete left half of the s-plane is the stability region. For commensurate FO LTI systems, stability region is a function of order α. A FO LTI system is stable if and only if eigenvalues of system matrix in the complex plane lie outside the angular sector defined by angles $\pm\alpha\pi/2$ [2]. From the point of view of stability, the range of α is divided into two parts i.e. $1<\alpha<2$ and $0<\alpha<1$. For the first case, stability region is a convex set while for the second case, it turns out as non-convex and thus the two cases are dealt differently. Stability of FO LTI system using root locus technique is investigated in [3], while BIBO stability aspect is addressed in [4]. Stability properties, modeling issues, controllability and observability aspects of FO systems have been discussed in [5–7]. Matignon's theorem is fundamental to many stability results for FO systems. Matignon's theorem is extended to the fractional systems with order $1<\alpha<2$ in [8]. Stability region in this case is convex set and thus LMI stability condition is directly derived. For the systems with order $0<\alpha<1$, stability region is non-convex, however LMI condition for the stability of such system is also given by identifying the instability domain [8,9]. LMI conditions for stability of the FO interval systems are derived in [10,11]. In view of latest developments, new proof of Matignon's theorem [7] valid for fractional order in the range $0<\alpha<2$ and independent of fractional derivative definition is given in [12].

In system modeling, frequency domain experiments are performed for system identification. From such experiments, many systems like heat conduction [13], transmission line model [14], biological systems [15] and financial systems [16] are found to exhibit fractional order dynamics and can be modeled more precisely using fractional order systems. Another electrical element called fractance, which has fractional impedance and properties intermediate between resistance and capacitance, is used to model the dynamics of the circuits more precisely [17]. Fractional order models for chaotic systems like Chua [18], Chen [19], Lü [20], Rossler [21] and Volta's [22] have also been developed over the period of time. Chaotic systems are special type of nonlinear systems which exhibit aperiodic oscillations. Minimum fractional order α for which chaotic behavior sustains is analytically derived in [23,24]. The problems of stabilization and synchronization of fractional order chaotic systems have been discussed in [25–28]. For chaotic systems, chaos synchronization is an important aspect especially from secure communication point of view and has been

* Corresponding author.
E-mail addresses: vivekmay12@gmail.com (V. Sharma), acdcmks@gmail.com (M. Shukla), bbs.iit@gmail.com, bbs.iit@gmail.com (B.B. Sharma).

https://doi.org/10.1016/j.chaos.2018.08.017
0960-0779/

studied in depth. In chaos synchronization slave system is made to mimic the behavior of master chaotic system. For fractional order systems, observer based chaotic system synchronization using scalar signal and its application to secure communication are discussed in [29,30].

To mimic and predict the behavior of real world systems, mathematical models based on integer or fractional order description are popularly used. However, performance of models deviate from the real systems, owed to the fact that real world systems have imperfections which are not modeled. Such systems are categorized as imperfect systems and have attracted the interest of researchers recently [31]. These imperfections stem from parasitic effects, inefficient manufacturing process and unknown perturbations [32]. These imperfections, called silent dynamics, play significant role in deciding the quality of the dynamics of the real systems. Design of observer to estimate the states of the imperfect system, without considering the effect of silent dynamics, will lead to inaccurate results. Imperfections in systems can be modeled as unknown input. Unknown input observer (UIO) is an important class of observers which can estimate the states of the system when system is excited by some unknown input [33]. Besides this, UIO find their application in robust fault detection [34] and secure communication [35]. UIO design was first proposed in [36] using geometric concepts. Necessary rank condition for the existence of observer, popularly known as observer matching conditions, are given in [37]. LMI conditions for the existence of UIO are derived in [38]. Design of full order FO UIO is proposed by Darouach in [39]. Theory of Unknown Input Observers is also extended to fractional order linear systems. Design methodology of UIO for FO LTI system is proposed in [40], wherein existence conditions for the proposed observer and LMI condition for the design of observer matrices are given. UIO for nonlinear systems using Lipschitz constant, one-sided Lipschitz constant and Differential Mean Value theorem (DMV) approach are given in [41–43], respectively. Design of FO UIO for Lipschitz class of nonlinear FO system is discussed in [44]. In this work, using Lyapunov direct method, LMI condition for the existence of observer for $0 < \alpha < 1$ is given. However, the proposed approach has the limitation that the complete stability region, which is non-convex for the considered range of order α, is not exploited and the approach as such can not be extended for order in range $1 \leq \alpha < 2$. These issues are addressed in present work by the use of DMV theorem. Using DMV theorem, error dynamics with nonlinear terms is converted to LPV dynamics and then LMI conditions, which exploit the complete stability region, are used to derive the proposed results. Moreover DMV based approach [43] is reported to be less conservative in comparison to Lipschitz based approach which is widely adopted in literature.

In present work, full order and reduced order FO UIO is proposed for the FO nonlinear systems driven by some unknown input. To account for the effect of nonlinearity, generally, Lipschitz condition is used to make the control design procedure tractable. However, in present work, nonlinear part in the error dynamics is expressed in appropriate equivalent form using DMV Theorem. It leads the error dynamics to evolve as LPV system. Stability of the error dynamics is then analyzed using convexity principles and corresponding LMI conditions are derived. Existence of the solution of LMI conditions guarantees the convergence of the error dynamics or in turn convergence of the observer states to the system states. Solution of the LMI gives the observer design matrices. The derived results are validated through extensive simulations by considering FO chaotic Chen and L*ü* systems which belong to the proposed class of nonlinear systems.

The paper is organized as follows: Preliminary results on stability of FO LTI systems are given in Section 2. Problem formulation for full order observer design for the proposed class of nonlinear FO systems is given in Section 3. In Section 4, main results of full order FO observer design are presented. LMI conditions for the existence of observer for both the ranges of order of the system i.e. $0 < \alpha < 1$ and $1 < \alpha < 2$ along with methodology to decouple the error dynamics from unknown input are derived in this section. Section 5 shows the simulation results to justify the proposed claim. Design of reduced order FO UIO with results of simulation is presented in Section 6 to Section 9. Finally conclusion of the work is drawn in Section 10.

Notations used. $\mathbf{b}_n(j)$ is a canonical basis vector of n dimensional space i.e. $\mathbf{b}_n(j) \in \mathbf{R}^n$ with all entries 0 except 1 at j^{th} position. *Co* means convex hull, D^α is the fractional derivative operator of order α.

2. Preliminary results on stability of FO LTI systems

Here, some basic definitions related to FO systems are given. Fractional order derivative is a generalization of integer order derivative. Fractional derivative is usually defined in following three ways [23].

The first is Riemann–Liouville derivative:

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \qquad n-1 < \alpha < n \tag{1} $$

The second way is the Caputo derivative:

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{\frac{d^n f(\tau)}{dt^n}}{(t-\tau)^{\alpha-n+1}} d\tau \qquad n-1 < \alpha < n \tag{2} $$

where $n \in \mathbf{N}$ and $\alpha \in \mathbf{R}^+$ and Gamma function $\Gamma(\cdot)$ is defined as

$$ \Gamma(\tau) = \int_0^\infty e^{-t} t^{\tau-1} dt $$

Caputo definition has the advantage of defining initial conditions for fractional order differential equations in the same way as for integer order differential equations.

Third way is the Grunwald–Letnikov (GL) definition:

$$ {}_aD_t^\alpha f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \begin{pmatrix} \alpha \\ j \end{pmatrix} f(t-jh) \tag{3} $$

where $[\cdot]$ means integer part.

In present work, Caputo definition is used for analysis of proposed class of systems. D^α is used to denote Caputo derivative of order α. Numerical approximation of the fractional derivative at instant kh is given as

$$ {}_{(k-L_m/h)}D_k^\alpha f(t) \approx h^{-\alpha} \sum_{j=0}^{N(t)} (-1)^j \binom{\alpha}{j} f(k-j) \tag{4} $$

where $N(t) = \min(k, L_m/h)$, L_m is the memory length, h is the time step of the calculation and $(-1)^j \binom{\alpha}{j} = C_j^\alpha \ (j = 0, 1, \dots)$ are binomial coefficients which are calculated iteratively as

$$ C_0^\alpha = 1 \qquad C_j^\alpha = \left(1 - \frac{1+\alpha}{j}\right) C_{j-1}^\alpha \tag{5} $$

If the FO LTI systems use commensurate order hypothesis i.e. if all the fractional orders in system are multiples of the same order then the following state space form is admissible

$$ {}_0D_t^\alpha \mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \qquad \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{6} $$