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Experimental hysteresis in memristor based Duffing oscillator

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ABSTRACT

This paper presents a practical implementation of a memristor based Duffing oscillator. We replace a diode based nonlinear element by a simple flux-controlled memristor in an existing circuit early proposed.We model the current-voltage characteristic and show experimentally that our circuit presents, for the same forcing amplitude, a route to chaos on the falling branch of the hysteresis loop, while its dynamics remains regular on the raising branch. This observation clearly highlights the memory effect of the memristor.

1. Introduction

The study of dynamical systems was marked over years and decades by three phenomenon: coherent structures, patterns and chaos. If the first phenomenon depends on the dispersion of the system, pattern formation focused on spatially extended systems and chaos depends on sensitivity to initial conditions. The common link of these ideas is the nonlinearity. Several research groups throughout the world have constructed nonlinear dispersive transmission lines using as nonlinear elements [1] either variable capacitance diodes [2–4] or saturating ferromagnetic inductors [5,6]. Chua [7] designed the first electronic chaotic circuit with a 3-segment piecewise-linear resistor as the nonlinear element (NLE). Since then, various electronic design or integrated circuits such as field -programmable gate arrays (FPGA) have been used for implementing nonlinear mathematical functions in chaotic systems [8,9].

The main idea in generating chaotic dynamics is to realize NLE. In this direction, one of the most important NLE is the memristor introduced in 1971 by Chua [10]. The memristor is a passive two terminal element establishing a relationship between the magnetic flux ϕ and the electric charge q. It can be charged-controlled, i.e $v(t) = M(q)i(t)$ or flux-controlled, i.e $i(t) = W(\phi)v(t)$, where $M(q) = \frac{d\phi}{dq}$ is the memristance and $W(\phi) = \frac{dq}{d\phi}$ the menductance. Thus, the memristor has the ability to memorize the past quantities of electric charges and magnetic fluxes. In 2008, researchers in Hewlett-Packard announced that a solid state implementation of the memristor has been successfully realized [11]. Subsequently, many applications of memristors in neuromorphic computation, neural networks, secure communications, signal processing, artificial intelligence, memristor based circuits to construct chaotic systems have attracted a lot of interest [12–19]. Just to cite a few, Itoh and Chua derived several nonlinear oscillators from Chua's oscillators by replacing Chua's diodes with memristors [20]. Sozen and Ucam provided a comparative analysis between low pass and high pass filter circuits using either ordinary resistor or memristor with a capacitor [21]. Muthuswamy realized a practical implementation of a memristor based chaotic circuit where the memristor was characterized by a cubic nonlinearity [22]. However, the above memristor circuits are costly as they are realized with electronic components such as operational amplifiers and analog multipliers. Furthermore, these electronic components require an external power supply to emulate the memristor, which unfortunately induces higher energy consumptions.

The Duffing oscillator, because of its clear physical meaning and easy implementation has been widely studied various mechanical or electrical models have been proposed [23]. It is known to exhibit chaotic behaviors under certain forcing frequencies or voltages. Recently Sabarathinam et al. have proposed an implementation of a memristor-based Duffing oscillator and investigated its nonlinear dynamics [24–26]. This investigation showed that the system clearly exhibits chaotic dynamics. However, in addition to the fact that this circuit needs an external power supply source, chaotic attractors obtained were basically different from classical Duffing oscillators. In contrast with the above approaches, we present in this paper an experimental system in which the memristor is a simple flux-controlled circuit proposed in [27,28], for

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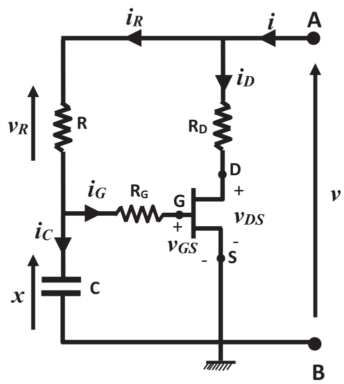

Fig. 1. Schematic diagram of the Biolek et al. emulator circuit.

which a bias voltage is unnecessary. The modelled equations of Biolek memristor circuit [27,28] are proposed. We also investigate the impact of memristor on the nature of the chaotic attractors, as well as the route to chaos on both the falling and raising branches of the hysteresis loop. Memristor based chaotic systems have been widely investigated in the literature [32–35]. However, to the best of our knowledge the nature of the dynamics appearing on hysteresis branches remains experimentally unexplored. The paper is organized as follows: In Section 2, we present the memristor circuit and the underlying equations; Section 3 is devoted to experimental results and the discussion; Section 4 ends the paper with a conclusion.

2. Modeling of memristor-based Duffing oscillator

2.1. The memristive device

The memristor used in this paper, as shown in Fig. 1 is implemented with a $2SK30A$ junction field effect transistor (JFET), three resistors R, R_G, R_D and a capacitor C. i_D, v_{GS} and v_{DS} are respectively the drain current, the gate-source bias and the drain-source bias. Its frequency analysis is performed by considering a sinusoidal voltage source $v(t) = V_0 \cos(\omega t + \varphi)$, where V_0 is the amplitude, φ is the initial phase and $\omega = 2\pi f$ is the pulsation with the frequency f. Fig. 2 shows the experimental $i - v$ characteristic. The obtained result depicts the well-known fingerprints of memristive systems: the pinched hysteresis loop. It appears in Fig. 2a and b that, the hysteresis effect strongly depends on the frequency. We can also observe that for each frequency, the characteristic is always pinched at the origin. The hysteresis lobe area decreases as the excitation frequency increases, which is one of the characteristics of the memristor circuit.

2.2. Mathematical modeling

The purpose in this section is to model the $i - v$ characteristic of the circuit in Fig. 1, That is to obtain the Duffing $i(v)$ nonlinear function. According to [27,29] the $n - th$ order voltage-controlled

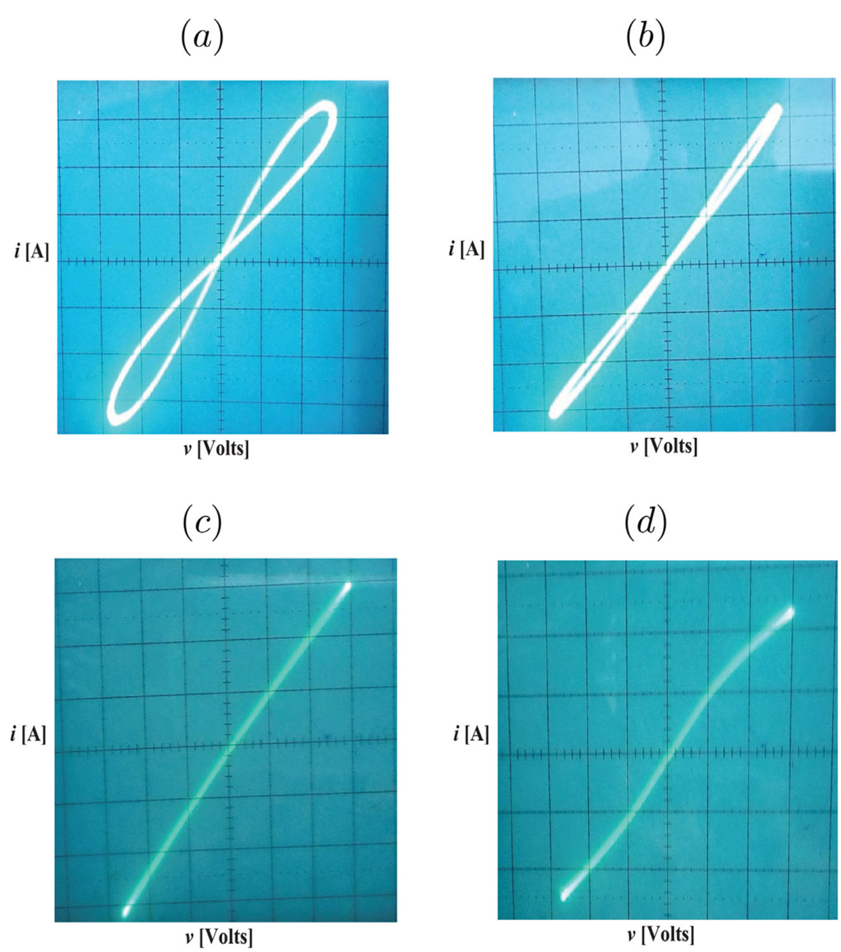

Fig. 2. Current-voltage characteristic of the memristor for $R_D = 1\,\text{k}\Omega$, $R = 1\,\text{M}\Omega$, $R_G = 10\,\text{M}\Omega$, $C = 100\,\text{pF}$ and $V_0 = 3\,\text{V}$: (a) $f = 1\,\text{kHz}$,(b) $f = 2\,\text{kHz}$, (c) $f = 10\,\text{kHz}$ ($R_G = 10\,\text{M}\Omega$) and (d) $f = 10\,\text{kHz}$ ($R_G = \infty$).