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Uncertainty and disturbance estimator based robust synchronization for a class of uncertain fractional chaotic system via fractional order sliding mode control

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ABSTRACT

This paper deals with a finite time robust synchronization problem of a class of uncertain fractional chaotic/hyper-chaotic systems with a novel fractional sliding mode control technique. Firstly, a fractional order sliding surface is proposed to mimic the behavior of master chaotic system. Then, a fractional order sliding mode control (FOSMC) methodology is derived analytically for convergence of all the synchronizing errors to zero in finite time. Finally, the derived control strategy is augmented with an auxiliary control based on uncertainty and disturbance estimator (UDE) for ensuring the robustness of the closed loop system dynamics in the presence of system uncertainties. Further, the uncertainties with unknown bounds are tackled for depicting the practical scenario and these results are also applicable to the N-dimensional uncertain chaotic as well as hyper-chaotic systems. Moreover, Mittag-Leffler and fractional order Lyapunov results are utilized to prove the stability and finite time convergence. Also, the proposed method delivers chatter-free control signal which is a major issue in sliding mode. MATLAB simulations are carried out to verify the efficacy and robustness of the derived results by considering two examples from literature.

1. Introduction

Fractional calculus (FC) has grabbed the great attention of research community worldwide [1–5]. Investigation in FC began with a question raised by mathematician L'Hopital from Leibniz about fractional differentiation in 1695. Since then, the fractional differential equations have been employed in modeling several real time systems [6–18] such as electrical circuits [6], viscoelastic beam [9], diffusion equation model [10], non-holonomic systems [11,12], chaotic systems [13–18] etc. Chaotic (with single positive Lyapunov exponent) or Hyper-chaotic systems (having two positive Lyapunov exponents) are special class of non-linear dynamical system that exhibit high sensitivity to parametric and initial condition variations [14]. Many ordinary chaotic systems have been redesigned as fractional order (FO) systems such as FO-Lorenz [15], FO-Chen [16], FO-Rossler [17], FO-Lu [18] etc. which can be termed as chaotic or hyper-chaotic systems.

The concept of chaotic synchronization was first explored from 1990's paper [19] by Pecora and Carroll. It is basically a master-slave chaotic configuration in which slave chaotic system is made to follow the master chaotic system dynamics. Wide variety of applications of chaos synchronization are found in secure communication [20], image processing [21] etc. Moreover, in recent years, the synchronization problem of fractional order chaotic systems has attracted the research fraternity. Various control techniques such as projective synchronization [22], adaptive control [23–26], passive control [27], active control [28], fuzzy based control [29], sliding mode control [30], feedback control [31], backstepping [33,34] etc. have been integrated with the chaotic dynamics to address their issues of synchronization as well as chaos suppression.

However, few shortcomings have been reported in previous investigations. The control techniques devised in [22,23,26–28,31,32,35,38,39] have not taken the system uncertainties into account. In [25,29,30,34,36,37,47], the system uncertainty with known bounds is reflected in one of the chaotic states and a single input control is created only for that particular system state. While this paper reduces such a design conservatism by considering unknown bounded complex uncertainties in every chaotic state. Furthermore, in most of the above works, the stability of the fractional order systems is ensured through conventional Lyapunov method, whereas, in this paper, the asymptotic stability has been proved through Mittag-Leffler and fractional order

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Lyapunov method [40–42] which renders more realistic approach. Moreover, the methods in most of the aforementioned papers, demonstrate that the convergence of the desired dynamics is guaranteed in infinite time. But, the results presented in this paper show the finite time convergence even in the presence of disturbances. The adaptive control approaches mentioned in the above studies can efficiently handle the parametric uncertainties, but combined numerical errors, sensor noise or unstructured uncertainties may drive system towards instability.

Among these control methods, the sliding mode control (SMC) is known for the best non-linear control technique for ensuring the robustness against system uncertainties [43]. But, the deficiency of this powerful method is chattering (high frequency oscillations) generated due to finite switching frequency operation or discontinuous action. Approximation functions such as tanh(.) or sat(.) reduces chattering as well as robustness properties. Therefore, to overcome this limitation, our proposed controller has introduced uncertainty and disturbance attenuator (UDE), developed by the authors in [44]. It has been applied to SMC in [45] for integer order linear time invariant systems (LTI). Since, the fractional order controllers provide better performances and robustness properties than their integer counterparts [47,48]. Thus, in this paper, the fractional order SMC is designed with a novel sliding surface. To the best of authors' knowledge, the proposed controller has yet not been developed in literature for such a class of system.

Inspired by the above description, this paper explores a new solution to synchronization problem of a class of uncertain fractional chaotic systems in master-slave configuration with a novel fractional sliding mode control technique. Firstly, a new fractional order sliding surface is proposed to mimic the behavior of master chaotic system. Then, fractional sliding mode methodology is derived for convergence of synchronizing error to zero in finite time. Finally, the formulated control strategy is augmented with the auxiliary control based on uncertainty and disturbance estimator (UDE) for ensuring the robustness of the closed loop system dynamics in the presence of system uncertainties. The uncertainties are introduced to the slave chaotic system and their bounds need not to be known in advance unlike prior works. These analytical results are applicable to the N-dimensional uncertain chaotic and hyper-chaotic systems. Moreover, the Mittag-Leffler and fractional order Lyapunov results are utilized to guarantee the stability and finite time convergence of closed loop dynamics. Also, the proposed method delivers chatter-free control signal, which is a major issue in sliding mode. MATLAB simulations are carried out to illustrate the efficacy and robustness of the developed scheme by considering two examples. First example is robust synchronization of two identical two-dimensional FO-Duffing Holmes chaotic systems inspite of the uncertainties and noise. Second example is synchronizing control of two different three-dimensional hyper-chaotic FO-Lu system and FO-Chen system with uncertainties.

The paper is arranged as follows: Section 2 describes the fundamentals of fractional calculus and stability concepts of fractional order control systems. Section 3 deals with the structure of fractional chaotic systems and its control problem. The proposed fractional order sliding mode controller for synchronization purpose is devised in Section 4. Section 5 provides the MATLAB simulation work with two examples. Eventually, the conclusion is drawn in Section 6.

2. Fractional calculus preliminaries

Fractional order calculus (FOC) is a generalized mathematical notion of integer-order calculus (IOC). Few main results relevant to FOC are described in the following sections:

2.1. Fractional derivatives and integrals

To deal with fractional order (FO) chaotic systems, we need to solve complex fractional differential equations (FDE). According to literature survey, there are three universally used definitions of FO differ-integrals: Grunwald-Letnikov, Caputo and Riemann-Liouville.

Elementary integro-derivative operator ${}_bD_t^{\gamma}$ is represented by:

$$ {}_bD_t^{\gamma} = \begin{cases} \int_b^t (d\tau)^{-\gamma} & \gamma < 0 \\ 1 & \gamma = 0 \\ \frac{d^{\gamma}}{dt^{\gamma}} & \gamma > 0 \end{cases} \tag{1} $$

where γ is a real order and (b, t) are lower and upper limits of integro-differential operation, respectively. The fundamental definitions are expressed as [1–3]:

1. Gruwald-Letnikov (GL) Definition:

$$ {}_0^{GL}D_t^{\gamma} = \lim_{h\to 0} \frac{1}{h^{\gamma}} \sum_{i=0}^{\infty} (-1)^i \begin{bmatrix} \gamma \\ i \end{bmatrix} f(t-ih) \tag{2} $$

2. Riemann-Liouville (RL) Definition:

$$ {}_0^{RL}D^{\gamma} f(t) = \frac{1}{\Gamma(m-\gamma)} \frac{d^m}{dt^m} \int_0^t \frac{f(\tau)}{(t-\tau)^{\gamma-m+1}}, $$
$$ \Gamma(m) = (m-1)!, t > 0, m \in Z, m-1 \le \gamma < m \tag{3} $$

where $\Gamma(\bullet)$ denotes Euler's gamma function defined by: $\Gamma(p) = \int_0^{\infty} t^{p-1} e^{-t} dt$

3. Caputo(C) Definition:

$$ {}_0^{C}D^{\gamma} f(t) = \frac{1}{\Gamma(m-\gamma)} \int_0^t \frac{f^m(\tau)}{(t-\tau)^{\gamma-m+1}} d\tau, m-1 \le \gamma < m \tag{4} $$

where m is a first integer, larger than γ. Note that the Grunwald-Letnikov and Riemann-Liouville derivatives are equivalent if $f(t)$ is a smooth function.

Further, the physical significance of fractional calculus is explained in [4] with interpretation of time and shadows on two planes. Various properties of fractional calculus are well presented in [5]. Lemma and some properties related to Riemann-Liouville (RL) derivative and Caputo (C) derivative are given as:

Lemma 1. The following inequality holds for an integrable function $f(t)$, if there exists atleast one $t' \in (0, t)$ such that $f(t') \neq 0$ then there is a constant $M > 0$ such that

$$ D^{-\gamma} |f(t)| \ge M \tag{5} $$

Property 1. For RL-Derivative, we have:

$$ {}_{t0}^{RL}D_t^{-\gamma} \left({}_{t0}^{RL}D_t^{\alpha} f(t) \right) = {}_{t0}^{RL}D_t^{\alpha-\gamma} f(t) - \sum_{i=1}^{m} {}_{t0}^{RL}D_t^{\alpha-i} f(t)\Big|_{t=t0} \frac{(t-t0)^{\gamma-i}}{\Gamma(1+\gamma-i)}, $$
$$ m-1 \le \gamma < m \tag{6} $$

Property 2. For RL and C-Derivative, we have the following equality:

$$ {}_{to}^{RL,C}D_t^{\gamma} \left({}_{to}^{RL,C}D_t^{-\beta} f(t) \right) = {}_{to}^{RL,C}D_t^{\gamma-\beta} f(t), \gamma \ge \beta \ge 0 \tag{7} $$

2.2. Stability analysis of fractional order non-linear systems

With the advent of fractional calculus, the stability analysis of non-integer non-linear systems has received a great attention