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Novel results on passivity and exponential passivity for multiple discrete delayed neutral-type neural networks with leakage and distributed time-delays

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ABSTRACT

This paper investigates the problem of passivity and exponential passivity for neutral-type neural networks (NNNs) with leakage, multiple discrete delay and distributed time-delay, via some novel sufficient conditions. Based on an appropriate Lyapunov-Krasovskii functional (LKF), free weighting matrix approach and some inequality techniques, enhanced passivity criteria for the concerned neural networks is established in the form of Linear matrix inequalities (LMIs). The feasibility of the attained passivity and exponential passivity criterions easily verified by the aid of LMI control toolbox in MATLAB software. Furthermore, we have compared our method with previous one in the existing literature, which depicts its less conservativeness. To substantiate the superiority and effectiveness of our analytical design, two examples with their numerical simulations are provided.

1. Introduction

In current scenario, extensive attention has been paid on neural networks (NNs) due to their fruitful applications in many fields, such as intelligent robot, signal processing, associative memories, fixed-point computations, automatic control, artificial intelligence, and so on [13,16,34,44]. It is well known that the time delays often occur from finite switching speed of the communication time, amplifiers and faults in the electrical circuits when the implementation of neural networks. Thus, the existence of time delays is inevitable in dynamical systems, which may affects the passiveness generating instability, bad performance, chaotic behavior and swinging or oscillation. Hence, the study on neural networks with time-delays have been considerable attention from many researchers, see for references [6,31,36]. In [43,46], the authors investigates the passivity criteria for time delayed neural networks with the help of LKF.

In accordance, the delay-dependent and delay-independent criteria are two categories of time delays in neural networks, which classified by the existing time delayed results. So far as, one can observe from available literatures, the delay-dependent case is less conserved than the delay independent ones. Further, time delay can be characterized into two types: a Discrete and Distributed delays. Here, we have taken both time delays, that is multiple discrete time delays and the distributed delays, into account while model our network system, because the length of the axon sizes are too large. For sequence, the passivity of various classes of neural networks with time delay has become an interesting area and different passivity conditions have been established for such NNs for multiple delays [53,57] and distributed time-delays [8,23,63]. As a result, it is noteworthy to inspect both the time-delay

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results in the passivity behaviors of systems, see for reference [18].

It is pointed out that a special case of time delay in neural networks that which incorporated to the negative feedback term of the system, namely forgetting or leakage terms. The present of this type of delays owing to some theoretical and technical difficulties. By the result of leakage delays the stability of the dynamical systems may affect and it leads to be unstable. So, over the past few years, time delay in leakage term has a enormous impact on the neural networks, see for references [12,24,26,38]. Thus, the consideration of leakage term in neural networks is necessary to passivity investigation [28,45]. The authors in [3], studied the stability of recurrent NNs when arise the leakage term in state variable via linear matrix inequalities. Mala et al., in [33], analyzed the problem of passiveness for neural networks, in which some suitable LKF are handled with leakage terms. Thus, the authors in [64], studied and obtained some results related to the problem for stability of time delayed neural networks with leakage terms.

Moreover, neutral time delay is another type of delay which has drawn a lot of attention nowadays. Because, a neutral-type delay phenomenon contains delays both in its derivatives and state variables. Practically, such phenomenon can be established in several fields including mechanics, automatic control, vibrating masses attached to an elastic bar, ecology, lossless transmission lines, robots in contact with rigid environments, heat exchangers, etc. Consequently, the delayed neural networks with neutral terms has received much attention in recent years and a enormous number of findings on this issue have been reported [2,10,27,37]. As for as, in [4], H. Bao and J. Cao discussed about the stability performance of neutral type neural networks with time delays. The less conserved criteria for stability of time delayed networks is obtained and investigated by Y. He, G. P. Liu, D. Rees in [14].

On the other hand, from the circuit theory, the concept of passivity was investigated and it plays a key role in the design and analysis of both linear and nonlinear systems, particularly for high-order systems. Also, the analysis of passivity has received a great attention [5,32,62], because it serves as a wonderful tool for studying the properties of nonlinear systems, such as signal processing, synchronization and stability. The core of the theory states that the properties of passiveness can ideally keep the system internally stable. This means that, the product of input and output in the passive neural networks is used as the energy provision and embodies the energy attenuation character which is a noteworthy feature of passivity. By the reason of its significance, the results on analysis of passivity for neural networks has been widespread studied in the available literature [9,25]. In [54], the authors examined the time dependent criteria for passivity in neural networks. In addition, P. Balasubramaniam, G. Nagamani and R. Rakkiyappan in [1], analyzed and obtained the LMI criteria for passivity, when the neutral and leakage delays are taken into account simultaneously. In current decades, the problem of passivity in the exponential sense for neural networks has been analyzed by many researchers. In [51], a new criterion were obtained for the addressed NNs to ensure the exponential passivity. It is note worthy to point out that, owing to its mathematical complexity, the exponential passivity condition has not been studied in the existing literature for neutral type neural networks with leakage, multiple discrete time-varying delay and distributed delay. Also, Du et al., investigates the existence of time delays in neural networks and exploit the LKF approach, the new sufficient criteria were derived for exponential passivity in [9]. So, the aim of this paper is meaningful and this motivates the present research problem.

Motivated by the earlier discussions, the problem for passivity and exponential passivity investigation of the neutral-type neural networks with leakage, multiple discrete delays and distributed time delays has not been fully studied yet, which motivates our current research work. With this aim to fill this gap, in this paper, we consider the passivity as well as exponential passivity problem for NNNs with leakage delays, multiple discrete delays and distributed delays. By utilizing the Lyapunov-Krasovkii functional, matrix theory and some inequality techniques, brand-new sufficient conditions for passivity and exponential passivity are formulated in the form of Linear matrix inequalities, which can be substantiated easily by LMI control toolbox in MATLAB software. As well, two numerical examples with their simulations are provided to illustrate the superiority and applicability of the proposed method. The main contributions of this work are highlighted as follows

(1) The neutral term, leakage time delay, multiple discrete delay & distributed time-delay are taken into account in the passivity and exponential passivity analysis of proposed neural networks.
(2) Based on different novel Lyapunov-Krasovskii functional, some sufficient conditions for passivity and exponential passivity of neutral-type NNs are derived in the form of LMIs. Furthermore, compared to the existing results, the derived conclusions are different and advanced.
(3) In this paper, the feasibility of the obtained LMIs for passivity and exponential passivity, easily solved by the aid of LMI control toolbox in MATLAB software.
(4) By handled the multiple discrete time-varying delay and neutral time-varying delay terms in our addressed neural networks, the allowable upper bounds of multiple discrete time-delay and neutral delay are maximum, when compare with the existing literatures, see Table 1, Table 2 and Table 3 in Example 5.1 & 5.2. This shows that the approach developed in this paper is effective and less conservative than some available results.

The organization of this paper is as follows. The considered time-delayed neutral-type neural network model is presented and some preliminaries are introduced in Section 2. In Section 3 & Section 4, by the Lyapunov-Krasovskii functionals and integral inequality techniques, some time delay dependent conditions are

Table 1
Maximum allowable upper bounds (h_k^*) of discrete delay for various τ_k.

Methods	$\tau_k = 0$	$\tau_k = 0.3$	$\tau_k = 0.5$	$\tau_k = 0.9$
In Ref [11]	-	0.5763	0.5679	0.5273
In Ref [22]	-	1.1921	1.1590	1.1081
In Ref [58]	-	-	1.4693	1.4243
In Ref [42]	5.346	-	4.592	-
In Ref [7]	8.7788	-	0.8912	0.3463
Theorem 3.1	28.71	21.627	19.39	14.4311

Table 2
Maximum allowable upper bounds (η_k^*) of neutral delay for different λ_k.

λ_k	0.2	0.5	0.9
In Ref [35]	-	4.5306	4.1632
In Ref [20]	-	4.7388	1.5708
Theorem 3.1	26.83	17.291	13.045

Table 3
Maximum allowable upper bounds (*MAUB*) h_k^* in Example 5.2 for different τ_k.

Methods	0.3	0.7	0.8	0.9
In Ref [30]	-	2.3368	1.9109	1.5523
In Ref [60]	-	2.0205	1.6377	1.2632
In Ref [56]	-	2.9436	2.5630	2.2150
In Ref [41]	1.4021	1.3916	-	1.3734
Theorem 4.1	22.59	17.104	10.221	7.28