Chaos, Solitons and Fractals 115 (2018) 300–305

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Frontiers

Mechanism of realizing a solitary state chimera in a ring of nonlocally coupled chaotic maps

E.V. Rybalova*, G.I. Strelkova, V.S. Anishchenko

Department of Physics, Saratov National Research State University, 83 Astrakhanskaya Street, Saratov 410012, Russia

ARTICLE INFO

Article history:
Received 27 August 2018
Accepted 3 September 2018

Keywords:
Nonlocal coupling
Chimera state
Solitary state
Multiplicative noise
Henon map

ABSTRACT

We study the mechanism of appearance of a solitary state chimera in a ring of nonlocally coupled two-dimensional chaotic maps. We show that both solitary states and the solitary state chimera can be induced by noise which multiplicatively modulates the nonlocal coupling coefficient in the ensemble of nonlocally coupled Henon maps. It is demonstrated that these states can also arise due to multistability in the network. It has been firstly shown and explained the possibility of obtaining the solitary states and the solitary state chimera in the one-dimensional ring of nonlocally coupled Henon maps. Basins of attraction of coexisting attractors essentially depend on the spatiotemporal structure, which is realized in the ring, as well as on the position of a particular oscillator among the other oscillators. We have found out that the riddling effect of the basins of attraction can be observed when the network state shows an amplitude chimera.

1. Introduction

Ensembles of interacting oscillators are widely distributed in the living nature and technique in the world around us. There are bird swarms, fish shoals, computer networks, power networks, the neural network of the brain, etc. Networks of nonlinear active oscillators with complicated individual dynamics are of special interest. Due to the collective interaction these systems show many various spatiotemporal structures as a consequence of nonlinear processes of self-organization [1–3]. When modeling the collective dynamics of complex ensembles, all elements are usually assumed to be identical, a specific model of the individual element is given, the coupling topology between elements, the ensemble dimension and possible external influences are determined and assigned. In the present paper we restrict ourselves to the case of nonlocal coupling between the elements of the considered ensemble. This implies that each element is connected symmetrically with P neighboring elements on its right and left sides and $1 < P < N/2$, where N is the total number of elements in the ensemble. If $P = 1$, the ensemble is characterized by local coupling. The case of $P = N/2$ corresponds to global coupling, i.e., each element is coupled with all ensemble elements.

It has been shown in [4–6] that spatiotemporal structures which are realized in ensembles of nonlocally coupled chaotic oscillators depend on the type of attractors of individual oscillators. It has been established that if elements have a pseudohyperbolic (quasihyperbolic) chaotic attractor [7], the regime of solitary states [8,9] can be observed in the ensemble. In this case individual elements are described by the Lorenz system [10] and the Lozi map [4]. When elements are characterized by a nonhyperbolic chaotic attractor, so-called chimera states [11,12] appear in ensembles. In this case the dynamics of individual nodes is determined by the logistic map [13], the Henon map [4], the Rössler system [14], the Anishchenko–Astakhov oscillator [15]. All these systems exhibit nonhyperbolic attractors [16].

The chimera state was discovered at the beginning of the century and represents a network state when all the network elements are divided into two spatially localized coexisting groups: one coherent (synchronous) and the other oscillating incoherently (asynchronously) [11,12]. Later on, this specific type of partial spatial synchronization has attracted great interest among experts in nonlinear dynamics and related research fields [13–15,17–27], since similar structures can be often encountered and observed in nature, e.g., Parkinson's disease [28], alternate sleeping of the brain hemispheres [29], partial synchronization in the brain neuronal activity with eye movement [30,31], the dynamics of the heart muscle [31], etc.

Solitary states are associated with a network regime when the majority of elements behaves coherently while certain sev-

* Corresponding author.
E-mail addresses: rybalovaev@gmail.com (E.V. Rybalova), strelkovagi@info.sgu.ru (G.I. Strelkova), wadim@info.sgu.ru (V.S. Anishchenko).

https://doi.org/10.1016/j.chaos.2018.09.003
0960-0779/

eral nodes demonstrate a completely different incoherent dynamics [8,9].

The paper [32] deals with a network consisting of two symmetrically coupled rings, where the ith element of the first ring is linked to the ith element of the second ring. Each ring represents an ensemble of nonlocally coupled discrete-time systems (maps). The elements of the first ring are described by Henon maps with nonhyperbolic chaotic attractors, while the second ring consists of nonlocally coupled Lozi maps with quasihyperbolic chaotic attractors. Our numerical studies have shown that when both rings are coupled, chimera states can appear in the ring of nonlocally coupled Lozi maps and solitary states can arise in the ring of nonlocally coupled Henon maps. It is important to note that these states can never be realized in the individual ensembles when there is no coupling between them. We have also revealed a novel type of chimera state, which appears in the Henon ring when it is coupled with the Lozi ring, and have called it a solitary state chimera (SSC) [32]. This structure is characterized by the appearance of a cluster (a region with distinct boundaries) which includes several solitary states and coexists with a coherence cluster and chimera states being typical for the ring of nonlocally coupled Henon maps.

The objective of the present paper is to find out the possibility of realizing the solitary state chimera in the isolated ensemble of nonlocally coupled Henon maps and to reveal and analyze the mechanism and conditions for its realization. Our numerical studies show that the appearance of the solitary state chimera almost follows the mechanism of emergence of solitary states [33]. Taking into account the fact that many networks, e.g., neural networks and other real-life ones, are functioning in noisy conditions, we also analyze the ensemble dynamics for the case when the nonlinear coupling coefficient is modulated by noise.

2. System under study

We analyze the dynamics of a ring of nonlocally coupled two-dimensional maps, which is described by the following system:

$$x_i^{t+1} = f(x_i^t, y_i^t) + \frac{\sigma}{2P}\sum_{j=i-P}^{j+P}[f(x_j^t, y_j^t) - f(x_i^t, y_i^t)], \qquad (1)$$
$$y_i^{t+1} = \beta x_i^t.$$

Here $i = 1, \ldots, N$ is the number of the individual element, $N = 1000$ is the total number of elements in the ensemble, P is the number of neighbors from the left and right sides, which each ith element is coupled with. σ and $r = P/N$ are the strength and the radius of nonlocal coupling, respectively. $f(x, y)$ and $g(x, y)$ are functions of a two-dimensional map which governs the dynamics of the individual element. In our research we choose the Henon map [34]:

$$x_i^{t+1} = 1 - \alpha(x_i^t)^2 + y_i^t,$$
$$y_i^{t+1} = \beta x_i^t; \qquad (2)$$

The control parameters of the map (2) are set as $\alpha = 1.4$ and $\beta = 0.3$ and correspond to the chaotic dynamics with a nonhyperbolic attractor in the individual element of the ring (1).

We explore the network (1) both without noise and in the presence of external noise when the coupling strength σ is modulated by a multiplicative noise source as follows: $\sigma = \sigma_0(1 + \sqrt{2D}\xi^t)$, where ξ^t is a random process distributed uniformly in the interval $[-1; 1]$, and D is the noise intensity. Initial conditions (x_i^0, y_i^0, $i = 1, \ldots, N$) are chosen randomly in the interval $[-0.5 : 0.5]$. We iterate the network (1) during $t = 5 \times 10^4$ in total and discard the first $t = 10^4$ iterations as a transient time.

As can be seen from (1), the individual oscillators are functioning in the nonautonomous regime, since they are forced by the signals due to their nonlocal coupling with P neighboring oscillators (the second term in (1)). This influence can be resulted in an essentially different dynamics of the individual oscillators as compared with the autonomous regime [35].

For the chosen values of α and β and for certain values of σ and r, the ring of nonlocally coupled Henon maps can demonstrate two different types of chimera states, namely, phase and amplitude chimeras [15]. These chimera states appear to be typical for networks consisting of nonlinear oscillators with nonhyperbolic chaos [5,6]. Ensemble elements belonging to an incoherence cluster of the phase chimera are characterized by a time-periodic dynamics and a random distribution of their phases. The amplitude chimera includes oscillators with a developed chaotic dynamics both in time and in space.

Fig. 1 exemplifies spatiotemporal structures which can be observed in the network (1) without noise ($D = 0$). A snapshot, that represents a spatial distribution of the instantaneous amplitudes x_i^t of all the network elements, is shown in Fig. 1a and illustrates the coexistence of incoherence clusters of amplitude ($18 < i < 170$) and phase ($360 < i < 402$ and $788 < i < 927$) chimeras with coherence domains. Fig. 1b depicts a space-time profile [15] of the ring dynamics, which represents a collection of the last 50 snapshots.

3. Noise-induced solitary state chimera

In the presence of external noise, the network (1) can exhibit various spatiotemporal modes which can be completely different from those observed without noise (Fig. 1). This situation is obvious since the individual oscillators in (1) are characterized by a high degree of multistability. The introduction of noise can affect the ensemble dynamics as strongly as the change in the initial conditions.

As was shown earlier in [5,6], the isolated one-dimensional ring of nonlocally coupled Henon maps cannot demonstrate solitary states and the solitary state chimera until it is coupled with the ring of nonlocally coupled Lozi maps. However, it would be interesting to find out whether these spatiotemporal structures can be induced by external noise in the isolated ring (1). In order to check this assumption, we modulate multiplicatively the coupling strength σ as follows: $\sigma = \sigma_0(1 + \sqrt{2D}\xi^t)$ and analyze the network (1) dynamics for different noise intensities. Numerical results are presented in Fig. 2. As follows from Fig. 2a, solitary states (in the interval $490 < i < 840$) appear and coexist with phase chimeras at $D = 0.0002$. For a larger noise intensity, $D = 0.000815$, the solitary state chimera ($705 < i < 845$) can be realized in the network (1) as shown in Fig. 2b. It is seen that the incoherence cluster with solitary states, i.e., the SSC, is formed between two phase chimeras.

The results presented in Fig. 2 prove that in the ensemble of Henon maps (1), the multiplicative noise can induce the regimes of solitary states (Fig. 2a) and the solitary state chimera (Fig. 2b) depending on the noise intensity D. It is worth noting that these spatiotemporal structures are revealed for the first time in the isolated network of nonlocally coupled Henon maps (1).

4. Mechanism of realizing the solitary state chimera

The effect of multiplicative noise induced transitions to the regimes of solitary states (Fig. 2a) and the solitary state chimera (Fig. 2b) testifies to the multistability of the system (1), which is caused by both the nonhyperbolicity of the Henon map and the influence of nonlocal coupling (the second term in (1)). In the presence of noise, phase trajectories intersect a separatrix which divides basins of attraction of different attractors. As a result, a part of the ensemble oscillators falls on one attractor but the other part is located on another one, as indicated by the results in Fig. 2. After falling on either attractor, the phase trajectory remains there and its return is possible only with an essentially large noise intensity.