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Optimal synchronization of non-smooth fractional order chaotic systems with uncertainty based on extension of a numerical approach in fractional optimal control problems

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ABSTRACT

In this paper, a control mechanism is presented for optimal synchronization of two non-smooth fractional order chaotic systems with parametric uncertainty based on nonlinear fractional order proportional derivative (NLFPD) controller combined with optimal periodic control signals. Unlike synchronization methods based on FPID controllers, in this approach, optimal tuning of the NLFPD controller and determination of optimal periodic control signals for synchronization process are presented in the form of non-smooth fractional order optimal control (FOC) problems. Optimal periodic control signals are demonstrated as a generalized expansion in the sense of Fourier expansion that is used to accelerate the synchronization process. Using the generalization of a numerical method in nonlinear optimal control problems and the Grunwald-Letnikov(GL) fractional derivative definition, the synchronization problem based on the suggested mechanism is transformed into the form of a smooth FOC problem. By defining a base-time soft switch, a supervisory approach is added to the proposed control mechanism for desired and robust performance against parametric uncertainty in the slave system. Finally, to illustrate the proposed control mechanism, the synchronization of two identical fractional order Chua systems with simulation results is presented. The Results show that using the suggested control mechanism, the synchronization is fast and robust against parametric uncertainty in Chua slave system.

1. Introduction

Fractional calculus was first introduced in the 17th century; hence it is an old mathematical topic. In fact, Fractional Calculus (FC) is the generalization and extension of integer calculus. Studying fractional derivatives became a very marvelous topic in mathematics field quickly; many different descriptions of fractional derivative operators were introduced, such as Riemann–Liouville(RL), Caputo, Grunwald–Letnikov(GL) [1–3] and in recent years, Cresson [4], Jumarie [5], or Klimek [6] and others [7]. Despite the long history of Fractional calculus, its application is elementary in various sciences and engineering fields. Various systems such as earthquake oscillation [8–9], wave [10] and chaotic equations in control engineering [11] are known to be displayed with fractional order dynamics. Also, the FOC problem is a branch of the fractional calculus of variations, which is a very interesting and active research area including nonlinear fractional order differential equations (FODEs) [12–14].There are two general approaches for solving optimal control (OC) problems including the indirect approach (solving fractional Euler–Lagrange equations or conditions of fractional Pontryagin-type) [15–18] and the direct approach (the problems which do not involve any necessary optimality Condition) [19–22]. When there are discontinuous functions or non-differentiable functions in dynamics of fractional and integer order system, the system is called non-smooth system. Because in direct and indirect methods, differentiation, Jacobin, gradient and Hessian of the system are required for determining the optimal solution [23]. The mentioned methods cannot be employed to solve OC and FOC problems. The necessary conditions for optimality based on non-smooth analysis are the major developments in non-smooth OC problems (see [24,25]), where usually, the non-smooth OC problems are converted to OC problems; although these optimality conditions cannot be used to solve the non-smooth OC problem in practical and numerical terms [23].The analysis and survey of nonlinear chaotic systems behavior with fractional dynamics is a new topic that has been widely investigated in this context such as in [26–29]. An important characteristic of chaotic systems is their extreme sensitivity to initial

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conditions. Accordingly, synchronizing or controlling chaotic systems is difficult. In [30], the fuzzy sliding-mode control (FSMC) for chaotic systems with uncertainty is proposed. An active sliding mode control method is proposed in [31] for synchronization of two chaotic systems with parametric uncertainty. Determining parameters of the active sliding mode controller for synchronizing two different chaotic systems is studied in [32]. An adaptive sliding mode controller is proposed in [33] to synchronize a class of drive– response chaotic systems with uncertainties and the backstepping control strategy for synchronizing the chaotic systems has been proposed in [34]. Due to applications of fractional-order chaotic systems in control processing recently, this system has attracted attentions increasingly. Especially in secure communication for instance in [35], the chaotic dynamics of the fractional order Genesio–Tesi system has been studied. Synchronization between a class of fractional-order chaotic systems and integer order chaotic systems based on sliding mode control using the stability theory of fractional order systems is proposed in [36]. To ensure existence of the sliding motion in finite time, an adaptive sliding mode method based on a novel fractional order switching type control law is presented in [37]. In [38], a modified method based on projective synchronization of fractional order chaotic systems with disturbance and unknown parameters is propounded. This method is based on the stability theory of fractional order systems and updated laws of the adaptive controllers and the parameters. Based on the input-to-state stable (ISS) theory, a single sinusoidal state coupling controller has been derived in [39] to achieve projective synchronization of a fractional order chaotic system.

Due to the many advantages of the fractional order controllers, including high flexibility, fast convergence speed and robust performance in most applications, the design and development of such controllers like fractional sliding mode controllers [40–42] etc has attracted attentions recently. Based on recent hardware developments and realizations of practical controllers, fractional order PID (FPID) controllers have been widely studied in [43] too. The concept of FPID controllers was first proposed in [44] by Podlubny. Due to high degree of flexibility, FPID controller gives more emancipation in controller design; accordingly, it is widely used by the designers. A type of NLFPID controller to enhance tracking properties of the given feedback system with a smaller design effort has been proposed in [45]. Another NLFPID controller is proposed for controlling the nonlinear processes in [46–48]. In this proposed controller, a widely nonlinear function can be used and also a classical form of the linear FOC by a 6 Degrees of Freedom (6DOF) in the mentioned controller is obtained. Furthermore, when this controller is used in the nonlinear FOC case, it is not necessary to use the nonlinear control techniques such as sliding mode control and etc. Among other features of the aforementioned controller, producing a large change in controller output, very small changes of control error in the case of linear FOC problem and creation of a smooth response which is more appropriate for the actuator, in the case of nonlinear FOC problem, can be mentioned [48]. In chaos synchronization fields, various types of PID and FPID controllers have been applied in [49–51]; [52] has presented a review on FPID controllers to provide more information on application of this controller. Various intelligent optimization methods such as particle swarm optimization (PSO) [53], evolutionary programming (EP) [54–56], modified ant colony optimization (ACO) [57] and etc, have been used for optimal tuning of PID controllers for chaos synchronization. Also, tuning of optimal FPID controller based on bacterial foraging algorithm is presented in [58] which demonstrates that this controller is more effective than the conventional PID controller for chaos synchronization.

Unlike most chaos synchronization based on integer and fractional order Chua dynamics(non-smooth fractional order chaotic system) such as Pecora–Carroll (PC) method, active–passive decomposition (APD) method, one-way coupling method, bidirectional coupling method [59,60], using robust observer [61] and other methods based on PID and FPID controllers, this paper has presented a supervisory control mechanism for synchronization of two identical Chua fractional chaotic systems with parametric uncertainty in the slave section. The proposed supervisory control mechanism is based on the NLFPD controller combined with periodic control signals. Optimal tuning of the NLFPD controller and determination of combined optimal periodic control signals for synchronization of Chua fractional order chaotic systems becomes a form of a non-smooth FOC problem. The proposed approach to solve this problem is based on generalization of a direct numerical method, presentation of a generalized expansion in the sense of Fourier expansion and using the GL fractional derivative, which ultimately transforms the non-smooth FOC problem into the form of the smooth nonlinear programming (NLP) problem. Simulation results of the master-slave fractional Chua oscillator based on the proposed supervisory control mechanism show that the synchronization process is done faster and with less control effort compared to some other known methods such as PC, APD, one-way coupling and bidirectional coupling. Furthermore, performance of the proposed control mechanism is desirable and robust against parametric uncertainty in the slave system.

This paper is organized as follows: existing definitions of FC and such theorems in approximation theory are represented in Section 2. Concepts of Master-slave synchronization between two identical non-smooth chaotic fractional order systems and expression of the used NLFPD controller with advantages and abilities are illustrated in Section 3. In section 4, a generalization of a numerical method in NLP problems for solving FOC problems, the best generalized periodic expansion in the sense of Fourier expansion and finally optimal synchronization problem with the supervisory strategy for robust performance, has been represented in a control mechanism scheme. At the end, the simulation results are demonstrated to investigate the implementation possibility of the proposed supervisory control mechanism in section 5.

2. Preliminaries

2.1. Fractional calculus

Among the most successful approaches in the field of fractional order operators, RL, Caputo and GL definitions are the most popular ones [7]. In this paper, we used the GL definition which is expressed as follows:

Definition 2.1 (Fractional Derivatives GL)**.** Let $f(.) \in C[0,t], t \in [a,b]$, $(n-1) \leq \alpha \leq n, h$ is the sampling time and $\Gamma(.)$,is the Euler's gamma function. The GL fractional derivative is defined by:

$$ {}_{a}^{GL}D_{t}^{\alpha} f = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^{j} \binom{\alpha}{j} f(t-jh), \tag{1}$$

where $[\cdot]$ expresses the integer part and $\binom{\alpha}{j}$ is the fractional binominal coefficient that defined as follows:

$$\binom{\alpha}{j} = \frac{\alpha!}{j!(\alpha-j)!} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)}. \tag{2}$$

This definition corresponds to RL definition of continuous time fractional derivative. Since this definition benefits from a discrete form, it has been used in most numerical calculations.