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Modeling the transmission dynamics of flagellated protozoan parasite with Atangana–Baleanu derivative: Application of 3/8 Simpson and Boole's numerical rules for fractional integral

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A multi-analysis of transmission dynamic of Trichomonas was undertaken in this study. A model describing the spread dynamic of flagellated protozoan parasite with a triadic considered. A sensitivity analysis study of parameters involved in the mathematical model with local differentiation is presented in detail using some statistical methods. Conditions for the existence and unicity of the exact solutions of the non-linear system are constructed and proofs presented in detail. New numerical methods based on the 3/8 Simpson rule and Boole's rule approach for the new fractional differentiation based on the generalized kernel is used to solve the transmission model with non-local operator of differentiation. Some numerical simulations are presented for different values of fractional operators. The numerical simulation of the modified model revealed that, the non-locality of fractional derivative could be used as uncertainties analysis. This explains why the fractional differentiation is a powerful mathematical tool for modeling real world problems.

1. Introduction

The spread of infectious diseases among mankind have attracted attention of many scholars around the world due to their fatal effect. It is believed that, a part from death due to accidents and nature death, infectious diseases constitute a major threat to humanity. Several of these diseases occur everyday; some are curable while others are not seeing for instance cancer. Some of them are manageable like HIV, other are not. While the spread is a great concern of medical doctors and pharmacists, white these researchers devote their time to provide suitable medications to safe mankind, while they put in place strategics to manage those that are manageable, while they invest in developing new vaccines to protect mankind, mathematicians more precisely modelers are investigations the theory underpinning the spread for predictions. Mathematicians or modelers used mathematical tools to build a mathematical model that will be used to predict the evolution of the spread in a targeted population. The most used weapon in these studies is perhaps the concept of differentiation and integration. Researchers have already established two classes of differential operators including: local differential operators and non-local differential operators. The last class is in fashion nowadays as it was found adequate mathematics tools to modeling physical problems with complex properties. They can also be subdivided in three categories: fractional derivatives with non-local and singular kernel, fractional derivatives with non-singular and local kernel and fractional derivatives with non-singular and non-local kernel. The fractional non-singular or continuous kernels do not obey classical mechanic law including index-law, these properties give them the ability to perform as strong forces, therefore they are able to explain many physical problems taking place in different scales which known as crossover behavior. They have recently been introduced and have already attracted attention of many scholars as they are found to be very accurate in modeling complex real world problems [1–5]. Without any doubt they are the future of modeling. Therefore in this paper, we applied the non-local and non-singular kernel fractional derivative model the spread of Trichomonas. The spread infections are bacteria, viruses and parasites, which can spread or move from one person to another during the activity. In this study we shall investigate the case of Trichomonas. The mathematical equations describing the spread or transmission of this virus was previously studied in [6]. Their model was sub-divided into susceptible males $S_m(t)$, infected males $I_m(t)$, susceptible st woman $S_{f_1}(t)$, infected st woman $I_{f_1}(t)$, susceptible le women $S_{f_2}(t)$, infected le women $I_{f_2}(t)$. Based on their assumptions, they derived the following system of nonlinear differential

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equations:

$$\begin{cases} S_m'(t) = \rho\eta - \mu S_m(t) - \lambda_{f_1}(t)S_m(t) + \gamma I_m(t), \\ I_m'(t) = \lambda_{f_1}(t)S_m(t) - (\mu+\gamma)I_m(t), \\ S_{f_1}'(t) = (1-\rho)\eta - (\mu+\alpha)S_{f_1}(t) - \lambda_m S_{f_1}(t) + \gamma I_{f_1}(t), \\ I_{f_1}'(t) = \lambda_m S_{f_1}(t) - (\mu+\alpha+\gamma)I_{f_1}(t), \\ S_{f_2}'(t) = \alpha S_{f_1}(t) - \mu S_{f_2}(t) - \lambda_{f_2}(t)S_{f_2}(t), \\ I_{f_2}'(t) = \lambda_{f_2}(t)S_{f_2}(t) + \alpha I_{f_1}(t) - (\mu+\gamma)I_{f_2}(t). \end{cases} \tag{1}$$

In the above system, susceptible humans to belonging to the population via the birth rate η, a ratio of ρ being males and $1-\rho$ being st-women as no one is born le. $\lambda_{f_1}(t) = \frac{\beta_{f_1} I_{f_1}(t)}{N_{f_1}(t)}$ is the fraction representing the susceptible males to acquire Trichomonas as result of contact with tr-female with β_{f_1} the effective contact rate for Trichomonas transmission from female to male. $\lambda_{f_2}(t) = \frac{\beta_{f_2} I_{f_2}(t)}{N_{f_2}(t)}$ is the fraction representing the susceptible females to acquire Trichomonas as result of contact with tr-female with β_{f_2} the effective contact rate for Trichomonas transmission from male to female; α is the rate of conversion of st-woman to le-women; γ recovering rate and μ is the death rate. The analysis of equilibrium points has been investigated in [6]. In this paper we present using statistic methods uncertainties analysis of parametersinvolved in the model. In addition, we will investigate the existence and uniqueness of solutions using some fixed-method, then we will present the model within the scope of fractional differentiation and finally the numerical solutions.

2. Uncertainties analysis

We must mention that a mathematical model can be very complex with several parameters and can as consequence lead its relationships between input, which are involved parameters, and output, which is the numerical simulation, may be poorly understood. Thus, a mathematical model can be considered as a box where the values of parameters input are opaque function of its outputs. Commonly, some or almost all model with parameters inputs is subjected to sources of uncertainty which include errors of measurement, lack of information and poor or partial understanding of main forces and dynamic of the system. Thus this non-understanding poses a limit on our confidence in the numerical output of the mathematical representations of the model. In addition, mathematical models representing real world problems may be to deal with natural intrinsic variability of the dynamic system, which they are describing. Therefore to accurately replicate the observed fact into mathematical formulation and obtain a best replication, good modeling process will requires that modeler present an evaluation of the confidence in the model. Thus, a requirement of quantification of uncertainty in model results, and follows by an evaluation of how each single parameter will contribute to output uncertainty. The model under consideration here has several parameters that could each influence the result in this case, we will present the sensitivity analysis of each parameter using some statistical notions. The model in Eq. (1) possesses 7 input parameters that could influence the output, in this section; we will present the effect of each parameter to the output. We assume that all the rates are considered, as probabilities therefore must range from 0 to 1.

The above values are used to generate the following numerical simulations. The numerical simulation use here to solve the above equation is the Picard iterative method with the following iterative formula.

Table 1
Theoretical values for sensitivity analysis.

γ	μ	α	ρ	η	β	Parameters
0.9	0.7	0.4	0.4	0.8	0.8	Values
0.6	0.5	0.3	0.35	0.7	0.7	
0.3	0.2	0.2	0.3	0.6	0.6	

$$\begin{cases} S_m^{n+1}(t) = S_m^n(t) + \int_0^t \Big(\rho\eta - \mu S_m^n(y) - \lambda_{f_1}^n(y)S_m^n(y) + \gamma I_m^n(y)\Big)dy, \\ I_m^{n+1}(t) = I_m^n(t) + \int_0^t \Big(\lambda_{f_1}^n(y)S_m^n(y) - (\mu+\gamma)I_m^n(y)\Big)dy, \\ S_{f_1}^{n+1}(t) = S_{f_1}^n(t) + \int_0^t \Big((1-\rho)\eta - (\mu+\alpha)S_{f_1}^n(y) - \lambda_m S_{f_1}^n(y) + \gamma I_{f_1}^n(y)\Big)dy, \\ I_{f_1}^{n+1}(t) = I_{f_1}^n(t) + \int_0^t \Big(\lambda_m S_{f_1}^n(y) - (\mu+\alpha+\gamma)I_{f_1}^n(y)\Big)dy, \\ S_{f_2}^{n+1}(t) = S_{f_2}^n(t) + \int_0^t \Big(\alpha S_{f_1}^n(y) - \mu S_{f_2}^n(y) - \lambda_{f_2}^n(y)S_{f_2}^n(y)\Big)dy, \\ I_{f_2}^{n+1}(t) = I_{f_2}^n(t) + \int_0^t \Big(\lambda_{f_2}^n(y)S_{f_2}^n(y) + \alpha I_{f_1}^n(y) - (\mu+\gamma)I_{f_2}^n(y)\Big)dy. \end{cases} \tag{2}$$

Except the above theoretical parameters, we also suggest the following initial conditions: The initial value of susceptible males is 35, initial value of infected males 5, susceptible str-female 130, les-susceptible 15, infected str-female 30 and les-female 5. We also assume that the total number of males is 500 and females 700. The numerical solutions are replicate in Figs. 1–5 as numerical simulation for the first, second and third lines of the table, standard deviation and skewness respectively.

It is clear from the three figures that, a change in a given input parameter produces a different numerical simulation. One can see especially a very drastic change in susceptible lesbian density population. From the above sample, one can carry on by having the contribution of the mean solutions, the standard deviation, the skewness and other. But here we present the numerical simulation for the contribution of standard deviation and skewness. The numerical simulations are depicted in Fig. 4 for standard deviation and Fig. 5 for skewness.

With this in mind, it is important to recommend that; good prediction of numerical model must always be accompanied with uncertainties analysis of parameters inputs. In the next section, we present the conditions under which the exact solution can be obtained.

3. Analysis of model with fractional differentiation

The concept of fractional differentiation has been used model real world problems in the recent decades. Some researchers revealed that, this concept is suitable for portraying chaotic behaviors. Looking the multi-simulations obtained in Figs. 1–4 by varying the parameters inputs of the model, nevertheless when checking the numerical simulation of a system of fractional differential equation with order alpha for instance, one will notice that, by fixing input parameters, each value of fractional order leads to different numerical simulations. One can conclude but there is no proof for this, that, a given fractional order for fractional differential equation maybe be equivalent to a given set of input parameters for classical model. With this idea in mind, we shall revert the classical model to the concept of fractional differential with non-local and non-singular kernel that was introduced in [7–15].