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On the motion of a pendulum attached with tuned absorber near resonances

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ABSTRACT

In this paper, the motion of two degrees of freedom of a dynamical system consists of a simple pendulum attached with tuned absorber subject to harmonic excitation is investigated. The governing equations of motion are obtained using Lagrange's equations in terms of the generalized coordinates. The multiple scales technique (MST) is used to gain the solutions of the equations of motion up to the third order of approximation. Two resonance cases namely; main (primary) external resonance and the internal one have been investigated to obtain the modulation equations. The amplitude and phase variables are obtained to investigate the possible steady state solutions and stability conditions. Time histories of the dynamical motions are discussed and presented graphically at any instant. In addition, resonance curves are graphically presented. The steady state solutions are obtained and their stabilities are checked.

Introduction

The main aim of the nonlinear dynamics subjects is to deal with the systems varying with time while dynamics can be used to analyze the system behavior regardless its degree of complication [1]. The importance of this science is due to its numerous applications in different fields [2–4].

Many researchers investigated the behavior of vibrating systems e.g. [5–11]. In [5–7], the authors deal with the stability of a spring pendulum system in which it has a complex behavior under certain resonance conditions. The multiple scales technique (MST) [12] is used to investigate the approximate solutions of equations of motion (EOM). They concluded that, the perturbed solutions up to second order approximation lead to good agreement with the original system than the first order of approximation. In [8–10], the authors investigated the previous vibrating system when subjected to either single or multiple excitation forces. They used (MST) to establish the approximate solutions of EOM up to the third order.

On the other hand, multi degree of freedom (MDOF) systems are investigated when the pivot point of a damped spring pendulum moves on a Lissajous curve [13], a circular path [14] and an elliptic path [15]. The authors investigated the resonance cases and stabilized the modulation equations in framework of MST. The stability of the steady state solutions are studied using Routh-Hurwitz criterion [16]. In [17], the authors extended their mentioned works [13,14] when a rigid body hanged up with a spring in which the other point of the spring is fixed. The generalization of this work is included in [18] when the attached point of the spring moves on an elliptic path. The solvability conditions are obtained after eliminating the observed secular terms in the different approximate solutions. The time histories of the attained solutions are represented graphically to reveal the effect of the different physical parameters on the motion of the considered system. In [19], the author investigated the numerical solutions of the EOM of a rigid body pendulum using fourth-order Runge-Kutta algorithms [20], taking into consideration the motion of the supported point will be in a horizontal path and depends upon time.

The auto-parametric pendulum models play a good role in damping the vibrations occurring in building structures when the model is in resonance or near it. In general, these models consist of primary system represented by nonlinear oscillator and another secondary one represented by a pendulum [21]. In [22], the nonlinear problem of 2-DOF auto-parametric damped spherical pendulum is investigated under the influence of an external excitation at the suspended point. Eisa et al. [23] studied more complicated model that has 3-DOF of the nonlinear vibration response of a rolling ship under the action of external and internal parametric excitation. They classified the emerged resonance cases and studied the steady state solutions close to these cases. The dynamic response of an oscillator connected with a pendulum vibration absorber constituting an autoparametric system is investigated in [24].

In the current study, the motion of autoparametric pendulum

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attached with tuned absorber subjected to harmonic excitation is under investigation. The pendulum-supported point moves with constant angular velocity in an elliptic path. The simple pendulum and the absorber are forced to move in a vertical plane. Lagrange's equations are used to obtain the EOM that represent two nonlinear second order differential equations. MST is utilized to establish the approximation solutions up to the third approximation. The internal and external resonance cases are investigated to obtain the modulation equations. The steady state solutions are studied, plotted and discussed in view of the stability conditions. Computer programs are used to represent the obtained solutions graphically in order to describe the manner of the considered dynamical system under the influence of the applied forces and the different parameters on the motion of the considered dynamical model at any time.

The importance of the considered pendulum model is due to its great applications in different fields like physics and engineering applications based on vibrating systems such as shipbuilding and ships motion, structure shaking [2], flow induced vibration [16], rotor dynamics, pumps compressors and transportation devices [4]. Moreover, the pendulum vibration absorbers are widely used for reducing the level of vibrations of engineering structures (chimneys, television towers, bridges, tall buildings, antennas), auto-balancing shafts, and so on.

Description of the model

Let us consider the motion of a dynamical model consists of a simple pendulum of mass M with length ℓ and a nonlinear absorber of a mass m constraint to move in the longitudinal direction. Supposing that, the point Q (on the ellipse) corresponds to the point N (on the auxiliary circle b) and moves in an oval path with constant angular velocity Ω as shown in (Fig. 1). Let us consider that OY and OX denote to the horizontal and the vertical downward axes respectively with the origin O. Moreover, let φ denotes the angle enclosed between the vertical and the line directed to QA and u represents the displacement of the absorber mass from equilibrium position.

Our aim now is to obtain the equations of motion, therefore the planar motion is considered, in which after time t one can write the coordinates of the point Q in the form

$$x_Q = a\cos(\Omega t), \quad y_Q = b\sin(\Omega t). \tag{1}$$

where a and b are the semi minor and major axes of the elliptic path

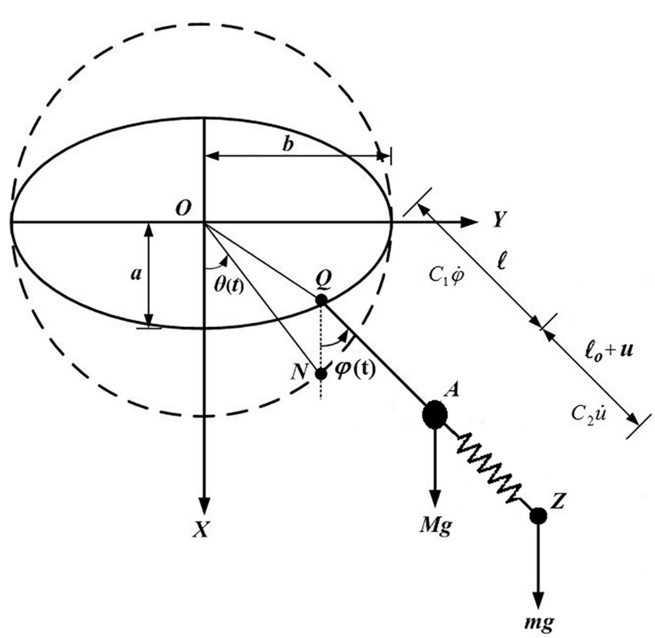

Fig. 1. The physical configuration.

respectively.

It is worthwhile to express, the kinetic and potential energies T and V of the model in the form

$$\begin{aligned} V &= \tfrac{1}{2}k_1u^2 + \tfrac{1}{4}k_2u^4 - Mg(a\cos\Omega t + \ell\cos\varphi) \\ &\quad - mg[a\cos\Omega t + (\ell + \ell_0 + u)\cos\varphi], \\ T &= \tfrac{1}{2}M[(b\Omega\cos\Omega t + \ell\dot{\varphi}\cos\varphi)^2 + (a\Omega\sin\Omega t + \ell\dot{\varphi}\sin\varphi)^2] \\ &\quad + \tfrac{1}{2}m([b\Omega\cos\Omega t + \dot{u}\sin\varphi + (\ell + \ell_0 + u)\dot{\varphi}\cos\varphi]^2 \\ &\quad + [a\Omega\sin\Omega t - \dot{u}\cos\varphi + (\ell + \ell_0 + u)\dot{\varphi}\sin\varphi]^2), \end{aligned} \tag{2}$$

where k_1 and k_2 represent linear stiffness of the spring and nonlinear one, ℓ_0 denotes the initial absorber length and the over dots refer to the derivative with respect to time. Based on the above equations, one obtains Lagrange's function $L = T - V$ directly. Therefore, Lagrange's equations for the generalized coordinates φ and u take the forms

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) - \left(\frac{\partial L}{\partial\varphi}\right) = Q_\varphi, \\ &\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{u}}\right) - \left(\frac{\partial L}{\partial u}\right) = Q_u, \end{aligned} \tag{3}$$

where

$$\begin{aligned} Q_\varphi &= F(t) - C_1\dot{\varphi}, \\ Q_u &= -C_2\dot{u}. \end{aligned} \tag{4}$$

Here, C_1 denotes the pendulum damping coefficient while C_2 expresses the absorber damping coefficient, $F(t) = F^*\cos(\Omega_1 t)$ indicates the acting force in the rotation direction that can be represented as harmonic function in which F^* represents the amplitude force of the system and Ω_1 is the forcing frequency. In the framework of obtaining the desired equations of motion, the following parameters are used

$$\begin{aligned} &c_1 = \frac{C_1}{\ell^2(m+M)}, \quad c_2 = \frac{C_2}{m}, \quad \beta = \frac{m}{\ell^2(m+M)}, \quad W_1^2 = \frac{g}{\ell_c} \\ &\omega_1^2 = \frac{W_1^2}{1+\beta(\ell+\ell_0)^2}, \quad \ell_c = \frac{\ell^2(m+M)}{\ell(m+M)+m\ell_0}, \quad f = \frac{F^*}{\ell^2(m+M)}, \\ &\omega_2^2 = \frac{k_1}{m}, \quad \alpha = \frac{k_2}{m}, \end{aligned} \tag{5}$$

It is worthwhile to mention that ($\omega_i;i = 1, 2$) represent the natural frequencies, g is the gradational acceleration and ℓ_c is the equivalent length of the pendulum. Making use of (2)–(5), the equations of motion take the form

$$\begin{aligned} &\ddot{\varphi} + c_1\dot{\varphi} + W_1^2\left(\varphi - \frac{\varphi^3}{6}\right) + \beta\Bigg[(\ell + \ell_0 + u)^2\ddot{\varphi} + 2(\ell + \ell_0 + u)\dot{u}\dot{\varphi} \\ &\quad + gu\left(\varphi - \frac{\varphi^3}{6}\right)\Bigg] + \Omega^2\left(\frac{1}{\ell_c} + \beta u\right)\left[a\left(\varphi - \frac{\varphi^3}{6}\right)\cos\Omega t - b\left(1 - \frac{\varphi^2}{2}\right)\sin\Omega t\right] \\ &= f\cos(\Omega_1 t), \end{aligned} \tag{6}$$

$$\begin{aligned} &\ddot{u} + c_2\dot{u} + \omega_2^2 u + \alpha u^3 - (\ell + \ell_0 + u)\dot{\varphi}^2 - g\left(1 - \frac{\varphi^2}{2}\right) \\ &\quad - \Omega^2\left[a\left(1 - \frac{\varphi^2}{2}\right)\cos\Omega t + b\left(\varphi - \frac{\varphi^3}{6}\right)\sin\Omega t\right] = 0. \end{aligned} \tag{7}$$

The previous system of Eqs. (6) and (7) consists of two nonlinear differential equations from second order.

The proposed method

The main target of this section is to utilize the MST to obtain the approximate solutions of Eqs. (6) and (7) up to the third order of approximation. Therefore, we express both of φ and u in terms of a small parameter $0 < \varepsilon \ll 1$ as

$$\varphi(t, \varepsilon) = \varepsilon\gamma(t, \varepsilon), \quad u(t, \varepsilon) = \varepsilon\xi(t, \varepsilon), \tag{8}$$

According to MST, we can express the asymptotic solutions γ and ξ as a power series of ε in the form