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Free vibration of arbitrary-shaped laminated triangular thin plates with elastic boundary conditions

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ABSTRACT

This paper studies the free vibration of arbitrary shaped laminated triangular thin plates based on a modified Fourier series method. An arbitrary shaped triangular plate is mapped into a right-angled isosceles triangular plate with unit length by the coordinate transformation. By padding another right–angled isosceles triangular plate which is near zero thickness on the plate after transformation. The displacement functions are then generally expressed as the combinations of Fourier cosine series and supplementary functions introduced to eliminate the discontinuous or jumping phenomenon in the boundaries. The classical thin plate elasticity theory is employed to construct the energy expressions of the plate. The Rayleigh-Ritz method is used in the present method to obtain all the unknown series expansion coefficients. A number of results are presented to verify the convergence and accuracy of current solution method to laminated triangular thin plates with arbitrary shapes, different material parameters and different boundary conditions. Additionally, some new results are given as the benchmark for future research, which are based on the various boundary conditions and geometric dimensions.

Introduction

The laminated structure is increasingly used in structural designs in recent decades, because it has many advantages over traditional materials, including higher stiffness and strength-to-weight ratios, good fatigue resistance and noise reduction. As a significant structural element in engineering applications, triangular plates are widely used in marine structures, automotive industry, construction engineering and several other fields. Meanwhile, any polygonal plate structure can be studied as a combination of the triangular plate. Therefore, it is quite necessary to investigate the vibration characteristics of the laminated triangular thin plate of arbitrary shapes, and it also has very significant theoretical and practical values to develop an accurate and efficient approach for free vibration analysis of the laminated triangular thin plate.

There are many valuable methods on the vibration problem of isotropic triangular plates, such as Ritz method [1–7], characteristic orthogonal polynomials method [8], the algebraic polynomials method [9], pb-2 Rayleigh-Ritz method [10,11], differential quadrature method [12], modified Fourier series method [13] and Fourier expansion method [14]. Most of the above methods had adopted coordinate transformation to transform the triangular domain into a more regular region, such as square regions [3,4,10] or right-angled isosceles triangular regions [2,6,13]. However, the singularity which is often overlooked in the numerical simulation will be introduced when the general triangular domain is mapped onto the square domain. The process which maps the arbitrary shaped triangular domain onto the right-angled isosceles triangular domain is usually smoother because it is a linear transformation. Huang and Sakiyama [15,16] proposed an another method to transform the triangular region into the quadrilateral region which adds a near zero thickness layer to the original plate.

Meanwhile, there are also many contributions for the vibration analysis of composite triangular plates. Bambill et al. [17] using the trigonometric co-ordinate functions and the Rayleigh-Ritz method to obtain the fundamental frequency of triangular orthotropic cantilever plates. Nallim et al. [18] analyzed the free vibration of the anisotropic triangular plate with elastically restrained edges. They used a set of characteristic beam orthogonal polynomials to approximate the deflection of triangular plates. Quintana et al. [19] proposed a general Ritz method for the vibration analysis of laminated triangular and trapezoidal thick plates based on the first order shear deformation plate theory, and the non-orthogonal right-angled triangular coordinate was used to approximate the geometry of plates. Chakraverty and Pradhan [20] investigated the free vibration characteristics of triangular functionally graded thin plate with various classical boundary conditions

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based on the Rayleigh-Ritz method, the displacement functions of the plate were expressed as the simple algebraic polynomials and the triangular plate was transformed into the right angled isosceles triangular plate. Zhang et al. [21] developed an element-free IMLS-Ritz method to investigate the vibration characteristics of the carbon nanotube reinforced functionally graded triangular plate with in-plane stresses based on the first order shear deformation plate theory. Belalia [22] used a hierarchical finite element to study linear and non-linear vibration of isosceles triangular functionally graded plates.

Despite lots of studies of triangular plates have been carried out, it appears that there are few researches on the vibration analysis of arbitrary shaped laminated triangular thin plates with elastic boundary conditions. Most of the studies on triangular plates are usually restricted to the classical homogeneous boundary conditions (clamped, simply supported and free). Inspired by these limitations, the authors propose a modified Fourier series method to analyze the free vibration characteristics of the laminated triangular thin plate of arbitrary shapes and elastic boundary conditions. In this paper, a coordinate transformation is used to transform the arbitrary triangular region into the right-angled isosceles triangular region, and the artificial boundary spring technique is introduced here to simulate the boundary restraints. The energy expressions of the laminated triangular plate are constructed from the theoretical level. On this basis, this paper presents several results for free vibration characteristics of the laminated triangular thin plate with various shapes, elastic boundary conditions, anisotropic degrees, and lamination schemes, which have been compared with results calculated by the finite element solution.

Theory and formulations

Description of the laminated triangular plate model

A triangular laminated thin plate under elastic boundary restraints is shown in Fig. 1(a), the mid-surface displacement of triangular plates in the x, y and z directions are respectively represented by the symbols u, v and w. In this study, four types of artificial virtual springs are adopted to simulate arbitrary elastic boundary restraints, which are three groups of linear springs (k_u, k_v and k_w) and one group of rotational spring (K_w). The variation of elastic boundary conditions could be realized by varying the stiffness values of the virtual boundary spring [23–28]. For instance, by setting all the stiffness values of boundary springs to extremely large, the clamped boundaries of laminated triangular plates can be achieved. Conversely, the fully free boundary condition can be directly obtained when the stiffness values are set to zero uniformly. For the sake of brevity, we have determined that the layers of laminated triangular plates are made from the same composite, and the thickness of each layer is the same with others. θ is the angle between the x-axis of triangular plates and the material coordinate of the k-th layer, Z_k and Z_{k+1} express the distance between the bottom surface and the upper surface of the k-th layer to the reference plane.

Kinematics and stress relations

On the basis of the assumptions of Kirchhoff, the displacement of the laminated thin plate can be expressed as the following linear relationships [29].

$$\begin{aligned} U(x,y,z,t) &= u(x,y,z,t) - z\frac{\partial w(x,y,t)}{\partial x} \\ V(x,y,z,t) &= v(x,y,z,t) - z\frac{\partial w(x,y,t)}{\partial y} \\ W(x,y,z,t) &= w(x,y,t) \end{aligned} \tag{1}$$

in which u, v and w denote the mid-surface displacements. t is the time variable.

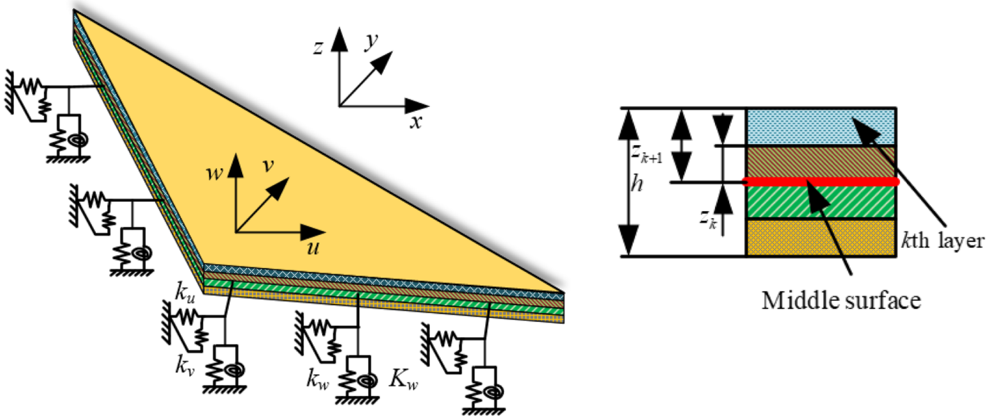
Based on the strain–stress relationship of elasticity theory, the linear strain–displacement relations of k-th layer for the thin laminated plates can be obtained:

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 + z_k\chi_x, \\ \varepsilon_y &= \varepsilon_y^0 + z_k\chi_y, \\ \gamma_{xy} &= \gamma_{xy}^0 + z_k\chi_{xy} \end{aligned} \tag{2}$$

in which ε_x^0, ε_y^0 and γ_{xy}^0 represent the normal and shear strains, χ_x, χ_y and χ_{xy} express the corresponding changes of bending and twist. In addition, z_k denotes the thickness of k-th layer. Then, relations of strain and displacement can be expressed as:

$$\begin{aligned} &\varepsilon_x^0 = \frac{\partial u}{\partial x}, && \varepsilon_y^0 = \frac{\partial v}{\partial y} \\ &\chi_x = -\frac{\partial^2 w}{\partial x^2}, && \chi_y = -\frac{\partial^2 w}{\partial y^2} \\ &\chi_{xy} = -2\frac{\partial^2 w}{\partial x\partial y}, && \gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{aligned} \tag{3}$$

According to the Hooke's law, the relation between stress and strain of laminated thin plates can be expressed as follow:



(a) (b)

Fig. 1. A thin laminated triangular plate: (a) coordinate system; (b) partial view.