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Path probabilities for consecutive measurements, and certain "quantum paradoxes"

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ABSTRACT

We consider a finite-dimensional quantum system, making a transition between known initial and final states. The outcomes of several accurate measurements, which *could be* made in the interim, define virtual paths, each endowed with a probability amplitude. If the measurements are *actually made*, the paths, which may now be called "real", acquire also the probabilities, related to the frequencies, with which a path is seen to be travelled in a series of identical trials. Different sets of measurements, made on the same system, can produce different, or incompatible, statistical ensembles, whose conflicting attributes may, although by no means should, appear "paradoxical". We describe in detail the ensembles, resulting from intermediate measurements of mutually commuting, or non-commuting, operators, in terms of the real paths produced. In the same manner, we analyse the Hardy's and the "three box" paradoxes, the photon's past in an interferometer, the "quantum Cheshire cat" experiment, as well as the closely related subject of "interaction-free measurements". It is shown that, in all these cases, inaccurate "weak measurements" produce no real paths, and yield only limited information about the virtual paths' probability amplitudes.

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1. Introduction

Recently, there has been significant interest in the properties of a pre- and post-selected quantum systems, and, in particular, in the description of such systems during the time between the preparation, and the arrival in the pre-determined final state (see, for example [1] and the Refs. therein). Intermediate state of the system can be probed by performing, one after another, measurements of various physical quantities. Although produced from the same quantum system, statistical ensembles, resulting from different sets of measurements, are known to have conflicting and seemingly incompatible qualities. These conflicts have, in turn, led to the discussion of certain "quantum paradoxes", allegedly specific to a system, subjected to post-selection. Such is, for example, the "three box paradox" [2–5], claiming that a particle can be, at the same time, at two different locations "with certainty". A similarly "paradoxical" suggestion that a photon could, on its way to detection, have visited the places it had "never entered, nor left, was made in [6,7], and further discussed in [8–11]. In the discussion of the Hardy's paradox [12–15] the particle is suspected of simultaneously "being and not being" at the same location [13]. The so called "quantum Cheshire cat" scheme [16–20] promises "disembodiment of physical properties from the object they belong to".

One can easily dismiss a "paradox" of this type simply by noting that the conflicting features are never observed in the same experimental setup, and therefore, never occur "simultaneously" [5,15, 21–24] . (We agree: one can use a piece of plasticine to make a ball, or a cube, but should not claim that an object can be a ball and a cube at the same time.) There have been attempts to ascertain "simultaneous presence" of the conflicting attributes by subjecting the system to weakly perturbing, or "weak" measurements [2,6,13,16]. However, such measurements only probe the values of the relative probability amplitudes, corresponding to the processes of interest [18,19,25] and by no means prove that these processes are, indeed, taking place at the same time. Furthermore, confusing these amplitudes with the value of a physical quantity, may lead to such unhelpful concepts as "negative numbers of particles" [13], "negative durations" spent by a non-relativistic particle in a specified region of space [26]. or "apparently superluminal" transmission of a tunnelling particle across a potential barrier [27].

Similar questions can be asked about macroscopic quantum systems, such as superconducting flux qubits [28]. The answers are often formulated in terms of the Leggett–Garg inequalities [29], which restrict the values of correlators of physical observables, under the assumption that macroscopic superpositions cannot persist for some fundamental reason (for a review see [30], and for recent developments [31]).

It is not our intention to compile an exhaustive list of relevant literature, or to discuss all aspects of the subject in great detail. The main purpose of this paper is to describe consecutive quantum measurements in a simple language, relying only on the most basic principles and concepts of elementary quantum mechanics. The brief introduction, already made, may have helped to convince the reader that such a description would indeed be desirable.

We start from a simple premise. With the initial and final states of the system fixed, there are many measurements which could, in principle, be performed in the interim. Connecting results of possible measurements, performed at different times, defines a *virtual* path, which a system could follow. For each virtual path quantum mechanics provides a complex valued probability amplitude $A(path)$. If the said measurements are actually made, the outcomes become a sequence of observable events, the path acquires a probability, $P(path) = |A(path)|^2$, and becomes *real*. (We use "real" as a natural complement to "virtual".) The set of all real paths, possible final destinations, and the corresponding probabilities, together define a classical statistical ensemble. We note that a similar ensemble could, in principle, be constructed also by purely classical means. For example, it is not difficult to imagine a gun, whose bullets would arrive at a point on the screen with exactly the same probability, as the electrons in the Young's double slit experiment. For someone, interested *only* in the number of particles per square centimetre, the two ensembles would be identical.

The quantum side of the discussion is, therefore, limited to the details of the ensemble's preparation. In the previous example, the same distribution can be achieved by aiming the gun at different angles, and varying the frequency of the shots, or by using quantum interference. In the following, we will be interested in exploiting the quantum properties of a system, and will return to a classical analogue only occasionally.