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Investigation of heat source reconstruction of thickness-through fatigue crack using lock-in vibrothermography

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ABSTRACT

Lock-in vibrothermography (LVT) is an active thermography nondestructive testing technique, which utilizes thermal contrast induced by lock-in modulated vibration to detect surface or subsurface defects. During the LVT testing, only the surface thermal distribution of the specimen can be directly captured. However, in most cases special cares need to be taken about the subsurface thermal distribution and the related heat source reconstruction. In this paper, a direct and inverse problem for vertical heat source reconstruction by using LVT is investigated. Specifically, a 3D transient analytical model is proposed to investigate heat source distribution of a metallic plate with a fatigue crack. The Tikhonov regularization is used to reconstruct the heat source from the analytical/experimental data. Further, *H*-index is introduced to quantitatively evaluate the reconstructed results. The shapes of reconstructed heat source from both analytical and experimental data are similar to a 'half-penny'. Additionally, the integral operator has a significant influence on the experimentally reconstructed result.

1. Introduction

Active thermography, as one of the competitive nondestructive testing methods, uses an external excitation (e.g. electromagnetic [1], optical [2] or vibration [3]) as a heating source to generate relevant thermal contrasts. The presence of defects from the abnormal thermal contrasts are then recorded by the infrared camera and further used for defect evaluation. Induction thermography has been found to detect the smallest cracks with a higher sensitivity compared to vibrothermography [4]. And by using advanced pattern extraction algorithms, this method can automatically detect and size the defects with a high accuracy [5,6]. However, this technique is limited to detecting the conductive materials. Optical thermography and vibrothermography extend active thermography to detecting the nonconductive materials. Compared to vibrothermography, optical thermography has the advantage of a higher repeatability and non-contact/non-invasive testing capability. Still, this method faces the problems of non-uniform illumination, low SNR and limited capability of depth detection [7]. The main advantage of vibrothermography is its selective heating for deep depth detection via the modulated (lock-in) excitation [8]. Lock-in vibrothermography [2,9] as a typical branch of vibrothermography, which employs high power lock-in excitation to vibrate the test specimen resulting in frictional heating, plastic and viscoelastic heating at the defect vicinity [10]. Owing to the sinusoidal-modulated excitation amplitude, the vibration is modulated which could reduce the impact damage to the tested sample [11]. With the development of this method, recent studies have been conducted for heat source reconstruction (source localization and shape reconstruction). Ouyang et al. [12] established a theoretical model to describe the relation between the surface temperature and the depth of cracks. Holland et al. [13] considered the crack heating as a series of arbitrarily located line sources and estimates the locations and intensities of each of these sources. Mendioroz et al. [14] theoretically demonstrated that the phase and the natural logarithm of the surface temperature as a linear function of the square root frequency. The slope of this linear relation is directly proportional to the defect depth. They further proposed a method combining experimental data and a stabilized inversion algorithm to reconstruct the shape and location of square flaws buried at different depths [15]. Recently, they extended this method to characterize vertical cracks of arbitrary shape [16]. The results show that very accurate reconstructions are obtained for shallow cracks. As the depth of the crack increases, the depth is precisely obtained although the shape of the crack is rounded and its area is slightly overestimated. The above model is based on the constant integral operator and thermal distribution. However, the time variant nature of thermal diffusion should be considered in practice because thermal diffusion changes thermal distribution and then influence the integral operator. Thus, a time dependent model with a proper regularization method is needed to

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achieve the accurate defect reconstruction.

Based on this objective, this paper proposes a 3D transient analytical model to investigate heat source distribution of a metallic plate with a fatigue crack. The Tikhonov regularization is used to reconstruct the heat source from the analytical data. Further, *H*-index is introduced to quantitatively evaluate the reconstructed results. The influence of integral operator on the experimentally reconstructed result is also discussed. The rest of this paper is organized as follows: Section 2 introduces the direct and inverse problem of heat source reconstruction, and Tikhonov regularization method; Section 3 discusses inversion of analytical (model-based) data; Section 4 discusses inversion of experimental data; and Section 5 presents the conclusion and proposes future work.

2. Theoretical background

2.1. Direct and inverse problem of heat source reconstruction

Generally, two problems are inverses of one another if the formulation of each involves all or part of the solution of the other [17]. For heat source reconstruction, either directly calculating the heat source based on thermal diffusion equation (direct problem) [18,19] or minimizing the difference between experimental result and model-based result (inverse problem) [20–22] can be used. In this paper, the direct problem is based on the given position and intensity of the heat source to solve the surface thermal distribution of the tested specimen; However, in most cases, the priori information of the above mentioned position and intensity are unknown. The inverse problem investigated in this paper aims to obtain the profile of the heat source based on the known surface thermal distribution.

2.1.1. Direct problem of heat source reconstruction

Fig. 1 shows an ideal crack frictional heat source in a plate. Specifically, a thickness-through crack is in a plate with a crack face Π. Both engagement force and frictional heating contribute to a heat source Ω which is normally smaller than crack face Π [23,24]. The area of the plate surface in X and Y direction is much larger than crack face, this plate could be regarded as a semi-infinite plate. The 3D transient analytical model to describe heat transfer problem can be described as following [25]:

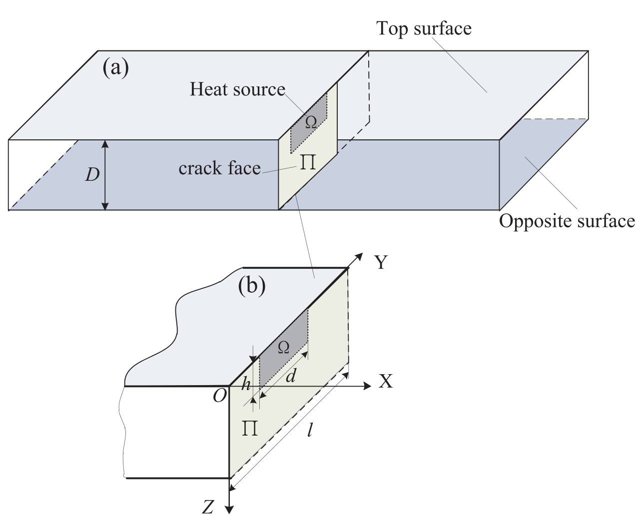

Fig. 1. The diagram of crack frictional heat source in a plate under ideal conditions. (a) Diagram of plate (b) 2.0× zoomed image of the crack face in (a). The thickness of the plate is *D*; the crack face is Π of $D \times l$; the frictional heat source is Ω of $h \times d$.

Table 1
Material property of the analytical model.

ρ (kg/m^3)	C (J/(kg·K))	λ (W/(m·K))
7750	480	50.2

$$T(x,y,z,t) = T_0(x,y,z) + \frac{\beta}{\lambda}\int_{\tau=0}^{t} d\tau \int_0^h \int_0^d G(x,y,z,t\,|0,\eta,\xi,\tau)Q(0,\eta,\xi,\tau)\, d\eta d\xi \tag{1}$$

where λ and $\beta = \lambda/\rho C$ are the thermal conductivity and thermal diffusivity. ρ and C are the specific heat and material density. $Q(0,\eta,\xi,\tau)$ is the heat power per unit area. $G(x,y,z,t\,|0,\eta,\xi,\tau)$ is the Image method based Green function:

$$G(x,y,z,t\,|\varepsilon,\eta,\xi,\tau) = \frac{1}{[4\alpha\pi(t-\tau)]^{3/2}}\sum_{n=-\infty}^{+\infty}\left\{\begin{aligned}&\exp[-\tfrac{(x-\varepsilon)^2+(y-\eta)^2+(z-\xi-2nD)^2}{4\alpha(t-\tau)}]\\&+\exp[-\tfrac{(x-\varepsilon)^2+(y-\eta)^2+(z+\xi-2nD)^2}{4\alpha(t-\tau)}]\\&+\exp[-\tfrac{(x-\varepsilon)^2+(y+\eta)^2+(z-\xi-2nD)^2}{4\alpha(t-\tau)}]\\&+\exp[-\tfrac{(x-\varepsilon)^2+(y+\eta)^2+(z+\xi-2nD)^2}{4\alpha(t-\tau)}]\end{aligned}\right\} \tag{2}$$

Then the temperature at any point on the plate surface is:

$$T(x,y,0,t) = T_0(x,y,0) + \frac{\beta}{\lambda}\int_{\tau=0}^{t} d\tau \int_0^h \int_0^d G(x,y,0,t\,|0,\eta,\xi,\tau)Q(0,\eta,\xi,\tau)\, d\eta d\xi \tag{3}$$

Thus, giving the position and power of the heat source, the spatial and transient thermal distribution of any point on the plate surface can be calculated by Eq. (3).

2.1.2. Inverse problem of heat source reconstruction

Normally, under the uniform heating condition the heat source Q can be expressed as:

$$Q(\varepsilon,\eta,\xi) = \Omega(\varepsilon,\eta,\xi)I \tag{4}$$

Here, Ω and I are the area and intensity (power) of the heat source. The calculation of thermal distribution needs the Fred Holm integral equation:

$$\begin{aligned}T(x,y,0,t) &= T_0(x,y,0) + \tfrac{\beta}{\lambda}\int_{\tau=0}^{t} d\tau \int_0^h \int_0^d G(x,y,0,t\,|\varepsilon=0,\eta,\xi,\tau)Q(\varepsilon=0,\eta,\xi,\tau)d\eta d\xi\\ &= T_0(x,y,0) + \tfrac{\beta}{\lambda}\int_{\tau=0}^{t} d\tau \int_0^h \int_0^d G(x,y,0,t\,|\varepsilon=0,\eta,\xi,\tau)\Omega(\varepsilon=0,\eta,\xi,\tau)Id\eta d\xi\end{aligned} \tag{5}$$

If the $\Delta T(x,y,0,t) = T(x,y,0,t) - T_0(x,y,0)$ is used, the Eq. (5) is rewritten as :

$$\begin{aligned}\Delta T(x,y,0,t) &= \tfrac{\beta}{\lambda}\int_{\tau=0}^{t} d\tau \int_0^h \int_0^d G(x,y,0,t\,|\varepsilon=0,\eta,\xi,\tau)\Omega(\varepsilon=0,\eta,\xi,\tau)Id\eta d\xi\\ &= A[\Omega(0,\eta,\xi,\tau)I] = A[Q(0,\eta,\xi,\tau)]\end{aligned} \tag{6}$$

Here, A is the integral operator [26].

In Eq. (5), the Q is accurate calculation result from Ω and I. However, in real situation the available data are often affected by a certain noise. Thus, the Eq. (6) is further expressed by:

$$A[Q^\delta] = I^\delta A[\Omega^\delta] \approx \Delta T^\delta \tag{7}$$

Here, δ is the noise level which meets $\delta^2 = \|\Delta T^\delta - \Delta T\|^2$.

In order to reconstruct the heat source, it becomes as a minimization