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Inelastic dynamic response of square membranes subjected to localised blast loading

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ABSTRACT

Extensive shock and highly localised blast waves generated by detonation of near field explosives (such as improvised explosive devices (IEDs)) are catastrophic to structures and humans, resulting in injury or death, progressive damage, or perforation through the structure and collapse. Mitigating the effects of such waves is paramount in various aspects of design engineering. A theoretical model is presented here to predict the large inelastic deformation of ductile thin square membranes induced by a generic, short pulse pressure load, comprising a piecewise function of spatial and temporal parts. Using the constitutive framework of limit analysis and incorporating the influence of finite displacements, two patterns of kinematically admissible, time dependent velocity profiles were investigated. These patterns included stationery and moving plastic hinges. The results were investigated in two cases: once with the interaction between bending moment and membrane forces retained in the analyses, and then when the response was solely governed by membrane forces.

For blast loads of high magnitude, the pressure was replaced by an impulsive velocity and the results were expressed in terms of dimensionless form of initial kinetic energy. The effects of boundary conditions and viscoplasticity have also been investigated. The theoretical results corroborated well with various experimental results in the literature, on ductile metallic plates such as high strength ARMOX steel and mild steel.

1. Introduction

Mitigating the catastrophic effects of high intensity localised blasts, such as those emanated from high explosives such as IEDs or buried land mines is of prime significance in the design of protective structures over the past decades. As the blast wave momentum varies exponentially with the stand-off distance (SOD) [1], near field charges give rise to localised response with much more focused energy release than those generated by distal ones, incurring large localised inelastic response and potential perforation of the structural element. As structural elements are most commonly designed as beams or plates, an assessment of the response for these elements subjected to blast waves is essential in the fields of civil, mechanical, military, and aeronautical engineering.

In fact, an extensive program of experimental and numerical studies has examined large deformation, damage evolution and failure of plates subject to uniform or localised impulsive loads. An extensive series of these studies are conducted by Nurick and co-authors [2–5] and by Børvik et al. [6–8], while the main thrust of theoretical work commenced after the seminal work of Hopkinson and Prager [9]. Following their research, the dynamic plastic collapse of the rigid, perfectly plastic plates of various characteristic dimensions have been investigated [10–15]. The studies on the dynamic response of circular plates subject to rectangular pulse pressure load established that the ratio of the blast duration to the total plate response is pivotal in idealisation of the blast with zero period, i.e. uniform momentum [16,17].

The dynamic response in terms of generalised deformations in a rigid, perfectly plastic structure is represented by evolution of plastic bending or shearing hinges. These hinges – either moving or stationery – are essentially discontinuity interfaces due to rotation (the bending hinge) or transverse shear strains (shearing hinge) leading to deformation localisations. At this weak discontinuity interfaces, the kinematic continuity of motion and the conservation of momentum must be satisfied. In thin membranes, the thickness is of a small order of magnitude compared to the characteristic in-plane lengths, the transverse shear forces may be assumed inconsequential as opposed to significant membrane forces. Thus, the deformation localisation is characterised by bending hinges only.

Langdon and Schleyer [18] presented a series of numerical, analytical and experimental studies on the connection characterization and pressure pulse response of the corrugated stainless-steel blast walls. They found that the yield pressure, i.e. the pressure to cause inelastic strains at the mid-point of corrugation was reduced by increasing

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Notations

The following symbols are used in this paper: Latin upper and lower case

$A_i - -D_i$	Integration constants [various]
$\bar{A}$	Elemental area [L^2]
B	Rectangular plate side width [L]
L	Rectangular plate side length [L]
$\dot{D}$	Internal energy dissipation rate [ML^2T^{-3}]
$\dot{E}$	External work rate [ML^2T^{-3}]
H	Plate thickness [L]
M	Plastic bending moment per unit length[MLT^{-2}]
M_0	Maximum plastic bending moment per unit length [MLT^{-2}]
N	Plastic membrane force per unit length [MT^{-2}]
N_0	Maximum plastic membrane force per unit length [MT^{-2}]
Q	Transverse shear force per unit length [MT^{-2}]
T_i	Duration of the ith phase [T]
T_f	Final time of motion [T]
V_0	Impulsive velocity of localised blast load [LT^{-1}]
W_i	Mid-point transverse displacement at the ith phase [L]
$\dot{W}_i$	Mid-point transverse velocity at the ith phase [LT^{-1}]
$\ddot{W}_i$	Mid-point transverse acceleration at the ith phase [LT^{-2}]
a	Loading coefficient [1]
b	Loading exponent [L^{-1}]
d_i	Ordinary differential equation constant [MLT^{-2}]
p_0	Maximum overpressure [$ML^{-1}T^{-2}$]
p_c	Static collapse pressure [$ML^{-1}T^{-2}$]
$p_1(x,y)$	Spatial part of pressure pulse load [$ML^{-1}T^{-2}$]
$p_2(t)$	Temporal part of pressure pulse load [1]
r_e	Loading constant (central) zone radius [L]
w	Generalised transverse displacement [L]
W_f	Mid-point transverse displacement [L]
z	Characteristic coordinate [1]

Greek lower case

α	Impulsive velocity parameter [L^2]
β	Static collapse pressure co-efficient [1]
ε_1	Impulse parameter [L]
ϕ	Characteristic angle [1]
η	Dynamic load amplification factor [1]
η_{crit}	Critical dynamic load factor [1]
ξ_0	Stationery plastic hinge length [1]
$\xi(t)$	Active travelling plastic hinge [1]
$\dot{\kappa}$	Curvature rate [T^{-1}]
λ	Dimensionless kinetic energy [1]
μ	Areal density (= ρH) [ML^{-2}]
ρ	Material density [ML^{-3}]
$\dot{\theta}_i$	Rotational (angular) velocity at the outer boundaries of zone i ($i = 1,\ 2$) [T^{-1}]
$\dot{\theta}_3$	Rotational velocity across the inclined plastic hinge [T^{-1}]
σ_0	Static plastic yield stress [$ML^{-1}T^{-2}$]
σ_{UT}	Ultimate tensile stress [$ML^{-1}T^{-2}$]
τ	Duration of the pulse [T]
ω_0	r_e/L [1]
ω_1	Pulse factor of pattern (A) [T^{-1}]
ω	Pulse factor of pattern (B) [T^{-1}]

the flexibility of the angle length connections. Nwankwo et al. [19] extended the theoretical analysis of the former authors to the CFRP (carbon fibre reinforced polymer) retrofitted blast walls using the elastic-perfectly plastic beam spring system with stationery bending hinges, but did ignored the membranal distortions at the retrofitted section of the wall. Zakrisson [20] conducted air blast and ground blast experiments on the localised response of simply supported Weldox 700E steel plate with PETN explosive. A smaller body of literature has examined the dynamic plastic response of two-dimensional beams, shells and thin membranes [13,21–23]. However, these were examined on stationery bending hinges with loading assumed to have a uniform distribution over the target; the transient phase associated with the travelling hinge has been neglected for simplicity in mathematical treatment. The purpose of this work is to examine the dynamic response of the plates with both moving and stationery bending hinges occurring due to localised blasts of various proximity of charge to the target.

Zheng et al. [24] investigated the elastic-plastic performance of stiffened square plates made of Q235 low carbon steel under confined blast. The confined blast was approximated with uniform distribution, leading to global deformation of the plate, while the deformation profile was unaffected by the stiffeners and no local buckling at the interface of stiffener and the plate was observed. Toolabi et al. presented a mixed finite element formulation to enrich the shear strain and deformations of Mindlin Reissner plate [25].

Most of the earlier studies were limited to classical theory of plates with infinitesimal deformations. The intensive shock wave may lead to large displacements in a thin plate brought about by geometry changes in contradistinction to the small deflection theory. When the deformation of the thin plate, due to a severe blast, is of a higher order of magnitude of its thickness, the structure would undergo finite displacements (geometry changes) which give rise to evolution of membrane (catenary) forces. The membrane forces so emerged will resist out-of-plane deformation and decrease its maximum at the cost of high in-plane tensile stresses. In some cases, experimental studies have revealed that in plated structures, exhibiting large deformation, the membrane forces dominate the overall performance [26–28]. A smaller number of theoretical research works in the literature has also catered for this phenomenon [13,29,30], however, these works have considered only uniform blast. Thus, there is a paucity of information due to rarity of systematic theoretical analysis on the permanent response of plates emanating from localised blasts.

Using this rationale, this paper derives and investigates the primary features of a theoretical solution for blast loaded thin square plates. Membrane forces are introduced as a part of the solution and emerge as deformations become finite.

This paper is, as such, an extension of previous works of literature [29,31,32] which dealt with applying the bound theorems of plasticity to derive explicit closed form theoretical solutions which catered for the problem of dynamic response in locally blasted square plates. Following this introduction, the general assumptions made throughout the study are presented. This is followed by a discussion of the governing equations and derivation of the static plastic collapse load in Section 3. In Section 4, a rigorous analysis is conducted on the dynamic performance of plates encompassing a wide range of loading and boundary conditions. For high magnitude pressure loads, the results are cast in terms of impulsive velocity in Section 5, where the influence of boundary conditions and strain rate sensitivity is briefly studied while the theoretical results are validated against available experimental and numerical results in Section 6. Finally, Section 7 presents and discusses the conclusions of the study.

2. Statement of the problem

2.1. Assumptions

The plates examined in this work are assumed to be ‘membranes’, implying that they are thin enough to render the contribution of transverse shear strains and rotatory inertia negligible. These effects are thus disregarded, while in-plane action plays a significant role in the overall