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Robustly stable adaptive horizon nonlinear model predictive control

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ABSTRACT

We present a new method for adaptively updating nonlinear model predictive control (NMPC) horizon lengths online via nonlinear programming (NLP) sensitivity calculations. This approach depends on approximation of the infinite horizon problem via selection of terminal conditions, and therefore calculation of non-conservative terminal conditions is key. For this, we also present a new method for calculating terminal regions and costs based on the quasi-infinite horizon framework that extends to large-scale nonlinear systems. This is accomplished via bounds found through simulations under linear quadratic regulator (LQR) control. We show that the resulting controller is Input-to-State practically Stable (ISpS) with a stability constant that depends on the level of nonlinearity in the terminal region. Finally, we demonstrate this approach on a quad-tank system and a large-scale distillation application. Simulation results reveal that the proposed approach is able to achieve significant reduction in average computation time without much loss in the performance with reference to fixed horizon NMPC.

1. Introduction

Model predictive control (MPC) has seen a great deal of success in the chemical industry, as it can naturally handle multiple-input-multiple-output systems with operating constraints. A survey of industrial applications of MPC is given in [16], and a thorough theoretical treatment of MPC is given in [21]. Nonlinear model predictive control (NMPC) has the added advantage of being able to capture nonlinear effects and thus provides higher accuracy across a wide range of states [5]. Fast NMPC implementations for large systems are enabled by noting that an exact solution of the associated nonlinear programming (NLP) problem is not necessary [14,28,25,27].

Terminal conditions are an important aspect of ensuring the stability of NMPC. However, calculating terminal constraints and costs for the nonlinear case is not straightforward. In [2], terminal conditions are calculated via the construction of a linear differential inclusion (LDI) and the solution of a linear matrix inequality. However, the construction of the LDI can be prohibitively difficult for large-scale systems.

In [1], a quasi-infinite horizon approach is proposed in which the terminal cost is computed based on a controller for the linearized system, and the terminal region represents a region of attraction for the linear controller applied to the nonlinear system. This method was applied to an experimental quad-tank system in [17] and further extended in [18]. Also, this method was developed for discrete time models in [10,19], which eliminates the need for a small discretization step upon implementation. These methods require finding a Lipschitz constant for the nonlinear part of the system or solving a series nonconvex optimization problems to global optimality, either of which makes application to a large system cumbersome. Instead, we propose a method of bounding only the higher order nonlinear effects of the system via simulations under linear quadratic regulator (LQR) control. This appears more practical and leads to a method of calculating terminal conditions that is scalable. Furthermore, we formulate terminal cost to approximate the infinite horizon problem rather than give it a strict upper bound. This is key for stability considerations later on.

We then consider another major issue in NMPC design, which is the selection of horizon length. In particular, we note a significant trade-off in this choice. The longer the horizon length, the larger the computational burden of the NLP that is solved online. In this case, control actions may be delayed, leading to degradation in control performance. The shorter the horizon, the smaller the region of the state space from which the terminal region is N-reachable. In this case, the problem solved online may become infeasible. Moreover, we recognize that this trade-off can vary with the state of the system. Thus, it is desirable to have a method for updating horizon lengths online. Typically, horizon lengths must be

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chosen to be conservatively long for practical applications in order to ensure feasibility of the dynamic optimization problem at every iteration. Furthermore, sampling intervals are limited by the worst case solve-time. Formulating an effective method for updating horizon lengths online is an important step towards overcoming these drawbacks of NMPC.

One method for updating horizon lengths is known as variable horizon MPC [22,23]. Here, the horizon length is treated as a decision variable in the optimization problem. However, in the nonlinear case, this leads to solving a mixed-integer nonlinear program (MINLP) online, which is currently impractical for large systems with significant nonlinearities.

Another approach is adaptive horizon NMPC (AH-NMPC), where the prediction horizon is updated online based on some rule. A special case of this approach is shrinking horizon NMPC [4], but in the more general case we allow for an expanding horizon as well as changes in the horizon of multiple step lengths. The methodology shown in [8] is also similar, except that our method does not necessitate knowledge of the required horizon length at a given time point. Instead, we propose a new method by which horizon lengths may be chosen in real time based on the current state. Regardless of how the finite horizon length varies from timepoint to timepoint, the terminal conditions serve to approximate the infinite horizon problem. We also propose a method that utilizes sensitivity updates from sIPOPT [15] in order to choose a sufficient horizon length in real time. We show that, under reasonable assumptions, AH-NMPC is Input-to-State practically Stable (ISpS).

In this work we combine the technologies of quasi-infinite horizon NMPC and adaptive horizon NMPC in order to provide a flexible NMPC formulation that retains stability properties with an adaptive horizon length. Finally, we demonstrate our methods on a quad-tank example and a large-scale distillation example.

2. Notation and definitions

We consider the system:

$$x_{k+1} = f_p(x_k, u_k, w_k) \tag{1}$$

with the model:

$$x_{k+1} = f(x_k, u_k) := f_p(x_k, u_k, 0) \tag{2}$$

where $x_k \in \mathcal{X} \subset \mathbb{R}^{n_x}$ is a vector of states that fully defines the model at time k, $u_k \in \mathbb{U} \subseteq \mathbb{R}^{n_u}$ is the vector of control actions implemented at time k, and $w_k \in \mathbb{W} \subset \mathbb{R}^{n_w}$ is the vector of disturbances that are realized at time k. Note that $\mathcal{X}$ is not a state constraint set to be added to the optimization problem, but rather the set on which the system is defined and ultimately the region of attraction of the controller. We use $|\cdot|$ as the Euclidean vector norm and $\|\cdot\|$ as the corresponding induced matrix norm. $\mathbb{R}$ is the set of real numbers, $\mathbb{Z}$ is the set of integers, and the subscript + indicates their nonnegative counterparts. Further, we assume that $f(x, u) : \mathbb{R}^{n_x+n+u} \to \mathbb{R}^{n_x}$ is twice differentiable in x and u with Lipschitz continuous second derivatives. We also make the following basic assumptions and definitions.

Assumption 1. (A) The set $\mathcal{X} \subset \mathbb{R}^{n_x}$ is control positive invariant for $f(\cdot,\cdot)$, that is, there exists $u \in \mathbb{U}$ such that $f(x, u) \in \mathcal{X}$ holds for all $x \in \mathcal{X}$. Furthermore, $\mathcal{X}$ contains the origin in its interior. (B) The set $\mathcal{X}$ is closed and bounded (C) The setpoint $(x_s, u_s) = (0, 0)$ satisfies $0 = f(0, 0)$. (D) The set $\mathbb{U}$ is closed and bounded, and contains zero in its interior.

Assumption 2. (A) The set $\mathcal{X} \subset \mathbb{R}^{n_x}$ is robustly positive invariant for $f_p(\cdot,\cdot,\cdot)$, that is, there exists $u \in \mathbb{U}$ such that $f_p(x, u, w) \in \mathcal{X}$ holds for all $x \in \mathcal{X}$, $w \in \mathbb{W}$. (B) The set $\mathbb{W}$ is bounded and $||\mathbf{w}|| := \sup_{k\in\mathbb{Z}_+} |w_k|$. (C) f_p is uniformly continuous with respect to w.

Definition 3. [Comparison Functions] A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{K}$ if it is continuous, strictly increasing, and $\alpha(0) = 0$. A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{K}_\infty$ if it is a $\mathcal{K}$ function and $\lim_{s\to\infty} \alpha(s) = \infty$. A function $\beta : \mathbb{R}_+ \times \mathbb{Z}_+ \to \mathbb{R}_+$ is of class $\mathcal{KL}$ if, for each $t \ge 0$, $\beta(\cdot, t)$ is a $\mathcal{K}$ function, and, for each $s \ge 0$, $\beta(s, \cdot)$ is nonincreasing and $\lim_{t\to\infty} \beta(s, t) = 0$.

Definition 4. [Stable Equilibrium Point] The point $x = 0$ is called a stable equilibrium point of (2) if, for all $k_0 \in \mathbb{Z}_+$ and $\epsilon_1 > 0$, there exists $\epsilon_2 > 0$ such that $|x_{k_0}| < \epsilon_2 \Rightarrow |x_k| < \epsilon_1$ for all $k \ge k_0$.

Definition 5. [Asymptotic Stability] The system (2) is asymptotically stable on $\mathcal{X}$ if $\lim_{k\to\infty} x_k = 0$ for all $x_0 \in \mathcal{X}$ and $x = 0$ is a stable equilibrium point.

Definition 6. [Lyapunov function] A function $V : \mathcal{X} \to \mathbb{R}_+$ that satisfies the following:

$$\alpha_1(|x_k|) \le V(x_k) \le \alpha_2(|x_k|) \tag{3a}$$

$$V(x_{k+1}) - V(x_k) \le -\alpha_3(|x_k|) \tag{3b}$$

for all $x_k \in \mathcal{X}$ where $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ is said to be a Lyapunov function for (2).

Theorem 7. *Under Assumption 1, if system (2) admits a Lyapunov function under some control law u_c, then (2) is asymptotically stable on $\mathcal{X}$.*

See Appendix B of [21] for the proof of the preceding.

We utilize the following properties in the case of plant-model mismatch.

Definition 8. (ISpS): Under Assumption 2, the system (2) is Input-to-State practically Stable (ISpS) on $\mathcal{X}$ if $|x_k| \le \beta(|x_0|, k) + \gamma(||\mathbf{w}||) + c$ holds for all $x_0 \in \mathcal{X}$ and $k \ge 0$, where $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}$, and $c \in \mathbb{R}_+$.

We note that Definition 8 is only useful given a reasonable bound on c. In the case that $c = 0$, Definition 8 simplifies to Input-to-State Stability (ISS). In Section 6.2, we show that c depends on the nonlinearities in the terminal region.

Definition 9. (ISpS Lyapunov function) A function $V : \mathcal{X} \to \mathbb{R}_+$ that satisfies the following:

$$\alpha_1(|x_k|) \le V(x_k) \le \alpha_2(|x_k|) + c_1 \tag{4a}$$

$$V(x_{k+1}) - V(x_k) \le -\alpha_3(|x_k|) + \sigma(|w_k|) + c_2 \tag{4b}$$

$$\forall \quad x_0 \in \mathcal{X}, \quad w_k \in \mathbb{W}, \quad k \in \mathbb{Z}_+$$

where $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$, $\sigma \in \mathcal{K}$, and $c_1, c_2 \in \mathbb{R}_+$, is said to be an ISpS Lyapunov function for (1).

Theorem 10. *Let Assumption 2 hold. If the system (1) admits a function $V(x)$ satisfying (9), then the system is ISpS.*

The reader is referred to [11] for more details on these definitions.

3. Nonlinear model predictive control

First we consider the traditional terminal cost/terminal region NMPC formulation:

$$\mathcal{P}(x) : V_N(x) = \min_{v_i} \sum_{i=0}^{N-1} L(z_i, v_i) + \psi(z_N) \tag{5a}$$

$$\text{s.t.} \quad z_{i+1} = f(z_i, v_i)\, \forall\, i = 0 \ldots N-1 \tag{5b}$$

$$z_0 = x_k \tag{5c}$$

$$v_i \in \mathbb{U}\, \forall\, i = 0 \ldots N-1 \tag{5d}$$