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Performance analysis of spectrum sensing schemes based on fractional lower order moments for cognitive radios in symmetric α-stable noise environments

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ABSTRACT

Spectrum sensing schemes based on fractional lower order moment (FLOM) are often used in impulsive noise environments in which traditional energy detectors are not applicable. The performance of FLOM-based detectors operating in an α-stable noise environment is difficult to evaluate. This is because α-stable random variables can usually only be modeled by the characteristic function since closed-form expressions are not available, except for the special values of the characteristic exponent that correspond to the Cauchy and Gaussian noise distributions. In this paper, we derive closed-form expressions for the probability density function (PDF) and corresponding complementary cumulative distribution function (CDF) for a symmetric α-stable random variable with an arbitrary characteristic exponent $\alpha\,(0 < \alpha \leq 2)$ in terms of the Meijer G–function. Consequently, we evaluate the receiver operating characteristics (ROC) curve for the FLOM detectors. The analytical results are validated with Monte Carlo simulations.

1. Introduction

The rapid growth in the demand for wireless services and the corresponding increase in the deployment of wireless networks in licensed and unlicensed frequency spectrum have placed severe burden on available spectrum. Cognitive radio (CR) technology has been shown to be a viable technology to minimize spectral holes and significantly improve spectrum utilization and efficiency [1]. In a CR system, unlicensed (secondary) users (SU) dynamically and opportunistically access frequency bands allocated to licensed (primary) users (PU) by continually monitoring the spectrum to identify when the PU is not using the spectrum or when it can be adequately protected from interference generated by the SU. A crucial first step in enabling CR systems is spectrum sensing [2]. By exploiting the spectrum in an opportunistic fashion, CR enables SU to sense which portions of the spectrum are available, select the best available channel, coordinate spectrum access with other users, and vacate the channel when a PU reclaims the spectrum usage right [3].

Several spectrum sensing techniques are available for detecting the presence of a PU. These can usually be classified into three categories: coherent detection, feature detection, and non-coherent detection [4–9]. Coherent and feature based detection schemes usually require knowledge of signal and noise characteristics of the PU in order to design the optimum detection scheme and, thus, yield improved detection performance. On the other hand, non-coherent sensing techniques do not require prior knowledge of the primary users signal characteristics. Because the transmission scenario of the PU is usually unknown to the SU, non-coherent spectrum sensing schemes are frequently adopted. The most commonly used non-coherent spectrum sensing technique is the energy detector [5–8,10]. The energy detector is useful not only because it usually produces reasonably good detection performance in Gaussian noise, but also because it is simple to implement [9]. However, the performance of the energy detector is limited by its sensitivity to noise uncertainty [11].

In the performance analysis of most schemes designed for spectrum sensing in CR, it is usually assumed that the noise background follows a Gaussian distribution based on a central limit theorem argument. However, in some practical CR environments, the noise may be non-Gaussian or impulsive in nature and spectrum sensing schemes based on the energy detector are known to be susceptible to severe performance degradation [4]. The energy detector, being a semi-blind procedure, is independent of the PU's signal properties but computes the sensing threshold based on knowledge of the noise statistics. Thus the accuracy of the energy detector depends on an accurate estimate of the noise power. Several generalizations of the energy detector have been introduced to

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improve its spectrum utilization in CR systems operating in non-Gaussian noise environments [12], [13]. In particular, replacing the squaring operation of the received signal envelope of the conventional energy detector with an arbitrary positive power p yields a more effective and more efficient spectrum sensing technique in a non-Gaussian noise and interference background [12]. Spectrum sensing schemes for which p is a fraction $(0<p<1)$, known as fractional lower order moment (FLOM) detectors, have been shown to be effective in non-Gaussian noise [13].

Several models are available in the literature to model impulsive aggregation of random effects. When the random effects have a heavy tail distribution, the sum of their effects follows a non-Gaussian stable distribution, of which the Gaussian distribution is a special case. Although the Gaussian model is symmetric about its mean and has finite variance, a non-Gaussian stable distribution may admit skewness and exhibit infinite variance and, in some cases, infinite mean. Stable distributions and stable processes are very useful in statistically characterizing the impulsive phenomena observed in a variety of systems [14]. The α-stable distribution has proved to be very successful in modeling practical noise sources, including both Gaussian and non-Gaussian noise with different degrees of non-Gaussianity through the selection of its characteristic exponent, α [15]. A stable law is a direct generalization of the Gaussian distribution and, in fact, includes the Gaussian as a limiting case. The main difference between the stable and the Gaussian distributions is that the tails of the stable density are heavier than those of the Gaussian density [16]. However, the practical difficulty in working with stable distributions is the lack of closed-form formulas for most stable symmetric distributions, except for the special cases of the Gaussian $(\alpha=2)$ and Cauchy $(\alpha=1)$ distributions [17].

Recently, a spectrum sensing scheme based on FLOM for detecting a primary user in non-Gaussian noise that can be modeled by symmetric α-stable (SαS) noise was proposed in [18,19]. Since the pth moment of an SαS is finite when $p\le\alpha$, the scheme proposed in [18] can be implemented in practice. However, the main drawback of the scheme is the lack of closed-form expressions for the statistical distribution of an α-stable random variable and, hence, the distribution of its pth moment does not exist in closed form. Consequently, the probability of false alarm and probability of detection for the scheme were based on Gaussian statistics although the test statistics is clearly not Gaussian. In this paper we perform the exact performance analysis of a FLOM detector operating in an SαS for an arbitrary value of α. Specifically, the main contributions of this paper are as follows;

1. We derive a closed-form expression for the probability distribution of an SαS random variable with rational values of the characteristic exponent $(\alpha=u/\nu)$, where u and ν are positive integers, in terms of the Meijer G-function. Consequently, the Cauchy $(\alpha=1)$ and Gaussian $(\alpha=2)$ distributions are obtained as special cases of the derived distribution.
2. Additionally, based on the derived distribution of the $S\alpha S$, we also derive the distribution of its fractional order moment (pth moment) in terms of the Meijer G-function.
3. Invoking the generalized central limit theorem of which the Gaussian central limit theorem is a special case, we derive the distribution of the test statistics under the hypotheses and consequently, derive expressions for the probability of false alarm (P_f) and probability of detection (P_d) of the test.

The remainder of the paper is organized as follows. An overview of the basic properties of SαS random variables is given in Section 2. Additionally, closed-form expressions for the PDF of an $S\alpha S$ random variable as well as that of its p-th moment are also derived in Section 2. In Section 3, the system model is presented and the distribution of the FLOM test statistics is derived. In Section 4, we analyze the performance of the FLOM detector. Graphical illustrations of numerical and computer simulation results are presented in Section 5. Finally, Section 6 is devoted to concluding remarks.

2. Alpha-stable distributions

The α-stable distribution has been used extensively to model impulsive phenomenon in many systems. In this section, we give an overview of some of the properties of an α-stable distribution which will be useful in the remainder of this paper. The α-stable distribution possesses several defining properties and characteristics [17], [20].

Property 1 (Stability Property). *If $X, X_1,$ and X_2 are independent α-stable random variables (r.v.) with the same distribution, then X can be expresses as a linear combination of X_1 and X_2, i.e.,*

$$a_1X_1+a_2X_2 \overset{d}{\Rightarrow} a_3X+a_4, \tag{1}$$

where $a_1, a_2,$ and a_3 are positive constants, $a_4\in\mathbb{R}$, and the symbol "$\overset{d}{\Rightarrow}$" implies that both sides have the same distribution.

Property 2. *Let the distribution of an α-stable random variable X be denoted as $S_\alpha(\beta,\gamma,\delta)$, i.e., $X\sim S_\alpha(\beta,\gamma,\delta)$, then for any $a\neq 0$, $b\in\mathbb{R}$,*

$$aX+b \sim \begin{cases} S_\alpha(\mathrm{sgn}\,(a)\ \beta,\ |a|\ \gamma,\ a\delta+b) & a\neq 1 \\ S_\alpha(\mathrm{sgn}\,(a)\ \beta,\ |a|\gamma,\ a\delta \\ +b-\frac{2}{\pi}a(\ln|a|\ )\gamma\beta) & a=1 \end{cases} \tag{2}$$

where $\mathrm{sgn}\,(x)$ is the sign (signum) of a real number x.

Property 3. *If $X_1\sim S_\alpha(\beta_1,\gamma_1,\delta_1)$ and $X_2\sim S_\alpha(\beta_2,\gamma_2,\delta_2)$ are independent, then*

$$X_1+X_2 \sim S_\alpha(\beta,\ \gamma,\ \delta) \tag{3}$$

where $\gamma=\left(\gamma_1^\alpha+\gamma_2^\alpha\right)^{1/\alpha}$, $\beta=\frac{\beta_1\gamma_1^\alpha+\beta_2\gamma_2^\alpha}{\gamma_1^\alpha+\gamma_2^\alpha}$, and $\delta=\delta_1+\delta_2$.

Property 4 (Tail of Distribution). *If $X\sim S_\alpha(\beta,\gamma,\delta)$, with $0<\alpha<2$. Then*

$$\lim_{x\to\infty}P(X>x) = C_\alpha\frac{1+\beta}{2}\left(\frac{\gamma}{x}\right)^\alpha, \tag{4a}$$

$$\lim_{x\to\infty}P(X<-x) = C_\alpha\frac{1-\beta}{2}\left(\frac{\gamma}{x}\right)^\alpha, \tag{4b}$$

where [17]

$$C_\alpha=\frac{1}{\int_0^\infty x^{-\alpha}\sin x\,dx} = \begin{cases} \dfrac{1-\alpha}{\Gamma(2-\alpha)\cos\left(\frac{\pi\alpha}{2}\right)}, & \text{if } \alpha\neq 1 \\ \dfrac{2}{\pi}, & \text{if } \alpha=1. \end{cases} \tag{5}$$

Note that, we may use the facts that

$$\Gamma(1-x)\Gamma(x)=\frac{\pi}{\sin(\pi\alpha)}=\frac{\pi}{2\sin\left(\frac{\pi x}{2}\right)\cos\left(\frac{\pi x}{2}\right)}, \tag{6}$$

to unify (5) as [20]

$$C_\alpha=\frac{2}{\pi}\Gamma(\alpha)\sin\left(\frac{\pi\alpha}{2}\right). \tag{7}$$

Property 5 (Fractional Order Moments). *The fractional order moment of the SαS random variable $X\sim S_\alpha(\beta,\gamma,0)$ with zero location parameter $(\delta=0)$ is given by*

$$\mathbb{E}\left[|X|^p\right]=\frac{2^{p+1}\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(-\frac{p}{\alpha}\right)}{\alpha\sqrt{\pi}\ \Gamma\left(-\frac{p}{2}\right)}\gamma^{\alpha/p}, \qquad p<\alpha. \tag{8}$$