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Robust identification of discrete-time linear systems with unknown time-varying disturbance

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ABSTRACT

In this paper, one robust identification method is proposed for the discrete-time linear systems with unknown time-varying disturbance. The disturbance is considered as a time-varying parameter for tracking estimation. A robust recursive least squares method is proposed using a forgetting factor. Moreover a new forgetting scheme to update the covariance matrix is developed to improve the stable convergence property of the time-invariant model parameters and the tracking performance of time-varying disturbance. The convergence performance of parameter estimation is analyzed with a proof. Two examples with different types of time-varying disturbance are shown to illustrate the effectiveness and advantage of the proposed method.

1. Introduction

When performing identification tests, the actual processes often encounter unknown disturbance [1–3], e.g. in a fuel cell system in the presence of time-varying disturbances [4]. The process sampled output data include unknown disturbance signals, which will cause undesired estimation error [5]. Some robust identification methods have become increasingly appealed in the recent years [6–8]. For practical applications, any identification methods should take into account the effect of ubiquitous time-varying disturbances and be as insensitive to disturbance changes as possible [9].

Although identification under time-varying load disturbances has attracted considerable attention, it has not been fully solved because of the unpredicted and unmeasured properties of such disturbances [10]. In discrete-time domain, a few robust identification algorithms were presented to deal with stationary stochastic noise for both open-loop and closed-loop identification tasks in the literature [11]. The error-bounded parameter estimation algorithm was proposed by using a membership set to deal with unknown but bounded disturbance [12]. Errors-in-variables methods were developed to obtain consistent estimation of the system input and output suffering from colored noises [13,14]. A filter technique was recently developed to estimate the Box–Jenkins model [15], using an auxiliary model to estimate the unknown noise-free output. A refined instrumental variable (RIV) approach was recently developed by using a unified operator to estimate the output error model subject to colored noise[16]. A bias compensation identification algorithm was proposed for ARMAX model subject to non-stationary disturbances in [17]. A robust estimation method was proposed by developing a filter to eliminate sinusoidal disturbances from sampled data [18]. By introducing sparse representations of the disturbances and using l_1-regularization with iterative reweighting to solve the sparse optimization problem, the cited ref. [19] presented a robust identification method for system in the presence of outliers and trends.

Based on the time-varying parameters estimation theory in [20], the unknown time-varying disturbance is considered as a slow time-varying parameter to estimate in this paper. The recursive least squares (RLS) method with forgetting factor has been widely used to track time-varying parameters [20–22]. The standard RLS algorithm uses a constant forgetting factor to compromise the performance among convergence rate, tracking, misadjustment, and stability, which may lead to the covariance 'wind-up' problem for poor excitation [23]. During poor excitations old information is continuously forgotten while very little new dynamic information comes in, which might lead to the exponential growth of the covariance matrix, and the estimator becomes extremely sensitive and susceptible to computational errors [24]. Thus some variable forgetting factor methods are proposed to avoid covariance 'wind-up' problem [24–26]. Since the system parameters are time invariant and the load disturbance parameter is time variant, the LS method with single forgetting factor scheme cannot realize which parameters cause the estimation error and gives poor estimation performances [25]. Thus some methods deal with models having time-varying parameters which change with different rates. One method is the directional forgetting scheme [26], which fixes

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one problem that the incoming information is not uniformly distributed over all parameters. Thus, the estimator wind-up can also occur when we estimate multiple parameters that each (or some) varies with a different rate. Therefore, a vector-type forgetting is proposed in [27] to assign different forgetting factors to different parameters and distribute incoming information uniformly over all parameters. By using a matrix pseudo-inversion lemma and an exponential weighting scheme, an extended recursive least-squares (ERLS) algorithm was developed for solving the over-determined normal equations in the instrumental variable approaches and estimation the time-varying model parameter [28]. By using two standard RLS methods and forgetting factors, an iteractive estimation strategy was used to identify a Hammerstein-type output error (OE) model or dual-rate sampled systems subject to load disturbance [29,30]. Inspired by the above methods dual forgetting factors are introduced to improve the tracking performance for time-varying disturbance.

In this paper, by extending the information vector, a robust recursive least squares (RLS) method is proposed, in which unknown time-varying disturbance is lumped into the model parameters for estimation. Considering that the disturbance is time-varying while the model parameters are time-invariant, a forgetting factors matrix is introduced to improve the convergence rates of estimating the model parameters and the load disturbance response, respectively. Base on the stochastic process theory, the convergence property of the proposed algorithm is analyzed with a strict proof. The paper is organized as follows. In Section 2, the identification problem is presented. The proposed algorithm is proposed in Section 3. The convergence analysis is also given in Section 4. Two examples are presented in Section 5. Finally, conclusions are drawn in Section 6.

2. Problem description

Considering the following discrete-time linear AutoRegressive eXogenous (ARX) system with unknown time-varying disturbance,

$$A(z^{-1})y(z) = B(z^{-1})u(z) + \eta(z) + v(z) \tag{1}$$

where $u(z)$ is the input excitation signal and generated by a zero-order hold and $\eta(z)$ is an unknown time-varying disturbance, which usually can not map the input–output causality and not be modeled with a constant parameterized noise model [4,17,30]. The polynomials $A(z^{-1})$ and $B(z^{-1})$ are coprime with the following forms,

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a} \tag{2}$$

$$B(z^{-1}) = b_1 z^{-1} + \cdots + b_{n_b} z^{-n_b} \tag{3}$$

Generally, the measurement noise $v(z)$ is assumed to be a Gaussian white noise with zero mean and unknown variance σ_v^2. $v(z)$ is uncorrelated with the input sequence, $u(z)$. The input signal sequences $u(z)$ and the output signal sequences $y(z)$ are assumed to be uniformly sampled with sampling interval T. The linear system is causal, i.e. $y(z)$ depends on $u(z_1)$ for $z_1 \le z$, but not on future values of $u(z)$ and $v(z)$.

Thus, the identification problem can be stated as: assuming that the degrees n_a and n_b are known. The identification objective is to estimate the linear ARX model parameters, $a_1, \ldots, a_{n_a}, b_1, \ldots, b_{n_b}$, by tracking estimation of the unknown time-varying disturbance.

3. Proposed algorithm

Firstly, denote the linear model parameter vector and observation vector, respectively, by

$$\theta_s = [a_1, \ldots, a_{n_a}, b_1, \ldots, b_{n_b}]^T \in \Re^{n_s} \tag{4}$$

$$\varphi_s(z) = \big[-y(z-1), \ldots, -y(z-n_a), u(z-1), \ldots, u(z-n_b)\big]^T \in \Re^{n_s} \tag{5}$$

where $n_s = n_a + n_b$ is the orders of the linear system model parameter.

For identification of the unknown time-varying disturbance, $\eta(z)$ is assumed to be a slowly drifting parameter to be estimated, and its excitation signal is considered as the one [17,29]. We further define the extended parameter vector and the corresponding extended observation vector as below.

$$\theta(z) = \big[a_1, \ldots, a_{n_a}, b_1, \ldots, b_{n_b}, \eta(z)\big]^T \in \Re^{n_e} \tag{6}$$

$$\varphi(z) = \big[-y(z-1), \ldots, -y(z-n_a), u(z-1), \ldots, u(z-n_b), 1\big]^T \in \Re^{n_e} \tag{7}$$

where $n_e = n_a + n_b + 1$ is the order of the extended vector.

Hence, the plant description in Eq. (1) can be rewritten as the following linear regression form,

$$y(z) = \varphi^T(z)\theta(z) + v(z) \tag{8}$$

Define the following cost function with a forgetting factor,

$$J(z, \hat{\theta}) = \frac{1}{2}\sum_{i=1}^{z} \lambda^{z-i}\big[y(i) - \varphi^T(i)\hat{\theta}(i)\big]^2 \tag{9}$$

Taking the first derivative of $J(z, \hat{\theta})$ with respect to $\hat{\theta}$, we have

$$\frac{\partial J(z, \hat{\theta})}{\partial \hat{\theta}} = -\sum_{i=1}^{z} \lambda^{z-i}\varphi(i)\big[y(i) - \varphi^T(i)\hat{\theta}(i)\big] \tag{10}$$

Letting Eq. (10) be zero, we obtain the least square estimation method

$$\hat{\theta}(z) = \left[\sum_{i=1}^{z} \lambda^{z-i}\varphi(i)\varphi^T(i)\right]^{-1} \times \left[\sum_{i=1}^{z} \lambda^{z-i}\varphi(i)y(i)\right] \tag{11}$$

By defining

$$P(z) = \left[\sum_{i=1}^{z} \lambda^{z-i}\varphi(i)\varphi^T(i)\right]^{-1} \tag{12}$$

and we have

$$\begin{aligned} P^{-1}(z) &= \sum_{i=1}^{z} \lambda^{z-i}\varphi(i)\varphi^T(i) \\ &= \lambda \sum_{i=1}^{z-1} \lambda^{z-1-i}\varphi(i)\varphi^T(i) + \varphi(z)\varphi^T(z) \\ &= \lambda P^{-1}(z-1) + \varphi(z)\varphi^T(z) \end{aligned} \tag{13}$$

Define the gain vector

$$K(z) = P(z)\varphi(z) \tag{14}$$

We can get a robust RLS method with a forgetting factor as follows

$$K(z) = \frac{P(z-1)\varphi(z)}{\lambda + \varphi^T(z)P(z-1)\varphi(z)} \tag{15}$$

$$P(z) = \frac{1}{\lambda}P(z-1)\big[\boldsymbol{I}_{n_m \times n_m} - K(z)\varphi^T(z)\big] \tag{16}$$

$$\hat{\theta}(z) = \hat{\theta}(z-1) + K(z)\big[y(z) - \varphi^T(z)\hat{\theta}(z-1)\big] \tag{17}$$