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Efficient recursive least-squares algorithms for the identification of bilinear forms

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ABSTRACT

Due to its fast convergence rate, the recursive least-squares (RLS) algorithm is very popular in many applications of adaptive filtering, including system identification scenarios. However, the computational complexity of this algorithm represents a major limitation in applications that involve long filters. Moreover, when the parameter space becomes large, the system identification problem is more challenging and the adaptive filters should be able to cope with this aspect. In this paper, we focus on the identification of bilinear forms, where the bilinear term is defined with respect to the impulse responses of a spatiotemporal model. From this perspective, the solution requires a multidimensional adaptive filtering technique. Recently, the RLS algorithm tailored for bilinear forms (namely RLS-BF) was developed for this purpose. In this framework, the contribution of this paper is mainly twofold. First, in order to reduce the computational complexity of the RLS-BF algorithm, two versions based on the dichotomous coordinate descent (DCD) method are proposed; due to its arithmetic features, the DCD algorithm represents one of the most attractive alternatives to solve the normal equations. However, in the bilinear context, we need to consider the particular structure of the input data and the additional related challenges. Second, in order to improve the robustness of the RLS-BF algorithm in noisy environments, a regularized version is developed, together with a method to find the regularization parameters, which are related to the signal-to-noise ratio (SNR). Furthermore, using a proper estimation of the SNR, a variable-regularized RLS-BF algorithm is designed and two DCD-based low-complexity versions are proposed. Due to their nature, these variable-regularized algorithms have good robustness features against additive noise, which make them behave well in different noisy condition scenarios. Simulation results indicate the good performance of the proposed low-complexity RLS-BF algorithms, with appealing features for practical implementations.

1. Introduction

In many system identification problems, the recursive least-squares (RLS) is the algorithm of choice. The main reason behind its popularity is its fast convergence rate, which is achieved even for highly correlated input signals [1–4]. From this point of view, it often outperforms, and by far, the family of least-mean-square (LMS) algorithms. On the other hand, the price to pay for this advantage is a significant increase in the computational complexity.

Several interesting solutions to reduce the complexity of the RLS algorithm (while preserving its fast convergence rate) can be found in literature. Among these, the dichotomous coordinate descent (DCD) method proposed by Zakharov and co-authors [5–7] is one of the most attractive approaches. This solution represents a computationally efficient alternative for solving the normal equations of the RLS algorithm. It does not need multiplications or divisions (these operations are simply replaced by bit-shifts), but only additions, so that it is well suited to hardware implementation. Consequently, the resulting RLS-DCD algorithm [6] was successfully applied in the context of different applications, e.g., see [8–17] and the references therein. Most of these applications are related to system identification problems, like echo cancellation, where the length of the impulse response is significant (e.g., hundreds of coefficients); therefore, the gain in terms of reducing the complexity is also important.

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Nevertheless, the system identification problems are more challenging when the parameter space becomes larger [18,19]. In the literature, such frameworks can be found in conjunction with different applications, e.g., [20–29]. For example, in the context of multichannel equalization [20], the coefficients of the channel and equalizer can be combined in a bilinear form. A similar form can be exploited in the context of nonlinear acoustic echo cancellation [21,22], where the nonlinear echo path is modeled by combining a linear filter with a nonlinear function, so that the global system resembles (to some extent) the Hammerstein model [30]. Also, similar approaches can be found in the context of target detection [23], system separability problems [24–26], source separation [27], multiple-input multiple-output (MIMO) communication systems [28,29], etc. Most of these approaches are related to the identification of bilinear (or trilinear) forms, based on tensor decomposition and modeling. In this manner, different problems of high dimension can be reformulated, so that low-dimension techniques are "tensorized" together [31,32]. Furthermore, the solutions can be found based on multidimensional adaptive filtering techniques.

In this paper, we focus on the identification of bilinear forms, while the extension to a higher dimension should be straightforward. In this context, the bilinear term is defined with respect to the impulse responses of a spatiotemporal model, which resembles a multiple-input/single-output (MISO) system. In [33], an iterative Wiener filter was developed for the identification of such bilinear forms. Solutions based on the LMS and normalized LMS (NLMS) algorithms can be found in [18,34], and [35], respectively. Recently, [36] provides an overview of these algorithms (including the RLS algorithm tailored for the identification of bilinear forms) and a detailed analysis of the LMS-based solutions. As shown in [36] and [37], the RLS algorithm for bilinear forms, namely RLS-BF, clearly outperforms its LMS-based counterparts in terms of convergence rate. However, the computational complexity of the RLS-BF algorithm is significantly higher.

Motivated by the appealing performance of the RLS-BF algorithm, the contribution of this paper is focused on two main directions. First, we aim to develop low-complexity versions of the RLS-BF algorithm based on the DCD method [5,6], taking into account the particular structure of the input data (specific to the case of bilinear forms), which brings additional challenges. Second, in order to improve the robustness of the system in noisy environments, we propose a regularized version of the RLS-BF algorithm. Moreover, the regularization parameters are iteratively adjusted so that the algorithm can behave well in different noisy conditions and especially against noise variations.

To this purpose, the rest of the paper is organized as follows. Section 2 provides a brief review of the bilinear model and the RLS-BF algorithm. In Section 3, two low-complexity versions of the RLS-BF algorithm are proposed, using the DCD method as an alternative way to solve the normal equations. Section 4 presents a regularized version of the RLS-BF algorithm, together with a method to select the optimal regularization parameters. Since the values of these parameters are related to the signal-to-noise ratio (SNR), a simple and practical way to estimate the SNR is presented in Section 5, which leads to a variable regularized RLS-BF algorithm; in addition, two low-complexity versions of this algorithm are developed (also based on the DCD method). Simulation results provided in Section 6 indicate the good performance of the proposed algorithms, which could represent appealing solutions for bilinear system identification problems. Finally, Section 7 concludes this work and outlines some perspectives.

2. Bilinear model and the RLS-BF algorithm

Let us consider the framework of the bilinear model from [36], where the reference signal is defined as

$$\begin{aligned} d(n) &= \mathbf{h}^T \mathbf{X}(n)\mathbf{g} + w(n) \\ &= y(n) + w(n), \end{aligned} \tag{1}$$

where n is the discrete-time index, the superscript ^{T} denotes the transpose operator, $\mathbf{h}$ and $\mathbf{g}$ are the two impulse responses of the system of lengths L and M, respectively,

$$\mathbf{X}(n) = \begin{bmatrix} \mathbf{x}_1(n) & \mathbf{x}_2(n) & \cdots & \mathbf{x}_M(n) \end{bmatrix}$$

is the zero-mean multiple-input signal matrix of size $L \times M$,

$$\mathbf{x}_m(n) = \begin{bmatrix} x_m(n) & x_m(n-1) & \cdots & x_m(n-L+1) \end{bmatrix}^T$$

is a vector containing the L most recent samples of the mth ($m = 1, 2, \ldots, M$) input signal, and $w(n)$ is the zero-mean additive noise. It is assumed that all the signals are real valued, and $\mathbf{X}(n)$ and $w(n)$ are uncorrelated. From (1), we can define the signal-to-noise ratio as

$$\mathrm{SNR} = \frac{\sigma_y^2}{\sigma_w^2}, \tag{2}$$

where $\sigma_y^2 = E\left[y^2(n)\right]$ and $\sigma_w^2 = E\left[w^2(n)\right]$ are the variances of $y(n)$ and $w(n)$, respectively, with $E[\cdot]$ denoting mathematical expectation.

As we can notice, the model from (1) resembles a simplified MISO system. The two impulse responses, i.e., $\mathbf{h}$ and $\mathbf{g}$, correspond to the temporal and spatial parts of the system, respectively. It is clear that for every fixed $\mathbf{h}$, $y(n)$ is a linear function of $\mathbf{g}$, and for every fixed $\mathbf{g}$, it is a linear function of $\mathbf{h}$. Thus, $y(n)$ is bilinear in $\mathbf{h}$ and $\mathbf{g}$ [38].

Based on the vectorization operation (i.e., conversion of a matrix into a vector [38]), the matrix $\mathbf{X}(n)$ of size $L \times M$ can be rewritten as a vector of length ML:

$$\begin{aligned} \mathrm{vec}\left[\mathbf{X}(n)\right] &= \begin{bmatrix} \mathbf{x}_1^T(n) & \mathbf{x}_2^T(n) & \cdots & \mathbf{x}_M^T(n) \end{bmatrix}^T \\ &= \widetilde{\mathbf{x}}(n). \end{aligned} \tag{3}$$

Therefore, the output signal $y(n)$ results in

$$\begin{aligned} y(n) &= \mathrm{tr}\left[\left(\mathbf{h}\mathbf{g}^T\right)^T \mathbf{X}(n)\right] \\ &= \mathrm{vec}^T\left(\mathbf{h}\mathbf{g}^T\right)\mathrm{vec}\left[\mathbf{X}(n)\right] \\ &= (\mathbf{g} \otimes \mathbf{h})^T \widetilde{\mathbf{x}}(n) \\ &= \mathbf{f}^T \widetilde{\mathbf{x}}(n), \end{aligned} \tag{4}$$

where $\mathrm{tr}[\cdot]$ denotes the trace of a square matrix, $\otimes$ is the Kronecker product, and $\mathbf{f} = \mathbf{g} \otimes \mathbf{h}$ is the spatiotemporal (or global) impulse response of length ML, which is the Kronecker product between the individual impulse responses $\mathbf{g}$ and $\mathbf{h}$. Consequently, the reference signal in (1) becomes

$$d(n) = \mathbf{f}^T \widetilde{\mathbf{x}}(n) + w(n). \tag{5}$$

The goal is to identify the temporal and spatial impulse responses $\mathbf{h}$ and $\mathbf{g}$ with two adaptive filters:

$$\widehat{\mathbf{h}}(n) = \begin{bmatrix} \widehat{h}_1(n) & \widehat{h}_2(n) & \cdots & \widehat{h}_L(n) \end{bmatrix}^T,$$
$$\widehat{\mathbf{g}}(n) = \begin{bmatrix} \widehat{g}_1(n) & \widehat{g}_2(n) & \cdots & \widehat{g}_M(n) \end{bmatrix}^T.$$